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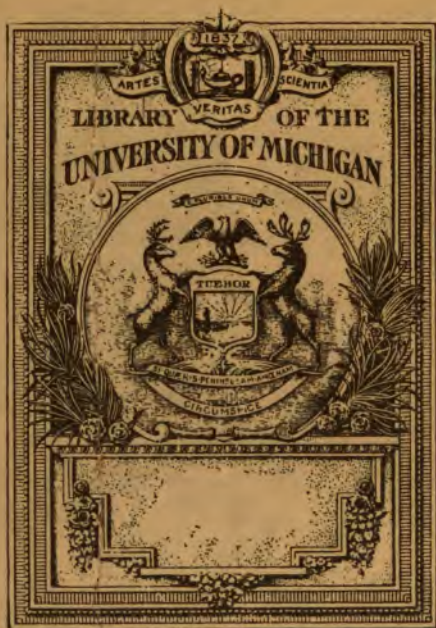
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DEFINITIONS IN PHYSICS



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DEFINITIONS IN PHYSICS

BY

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PREFACE

ABOUT six years ago the author wrote a pamphlet on "Definitions in Physics," which was intended mainly for his own students in the State University of Iowa. The small edition was exhausted in less than two years, since the pamphlet was used in several other universities. In spite of the frequent demands for a new edition, the publication of the book was delayed until it could be thoroughly revised.

To write a book of this kind is neither an easy nor a pleasant task. In most cases the only satisfactory definition of a physical quantity is the equation connecting it with other quantities previously defined. Its concept must be evolved by a backward process, and can become clear only after we have learned all about its measurement and application to physical problems. It will be found that in many such cases the author has not made any attempt to find the "deeper" meaning of some mathematical relation between two or more physical quantities, however important the relation might have become in our science.

Many writers of textbooks have tried to find a solution of the difficulty by giving to a new quantity a more familiar name, and thus have befogged the minds of the students. For example, the definition of difference of potential usually begins: "It is that work which —"; but we know that it is not work at all. The main object of this little book is to counteract the bad influence which loose statements of this kind have in the teaching of elementary physics. They lead to mathematical equations in which the dimensions on the two sides are not identical.

Frequently the same terms are used by different authorities with different meaning. In such cases a choice must be made which will arouse the criticism of those who prefer to use the term for something else.

Another matter left to the individual judgment of the author is the selection of certain fundamental concepts upon which he wishes to build up his system, for our science is nothing but an attempt to present in a systematized form the very numerous physical phenomena. The choice as to what should be considered fundamental remains more or less arbitrary. Somewhat against his better judgment the author has followed the customary method, except that the definition of mass has been deferred

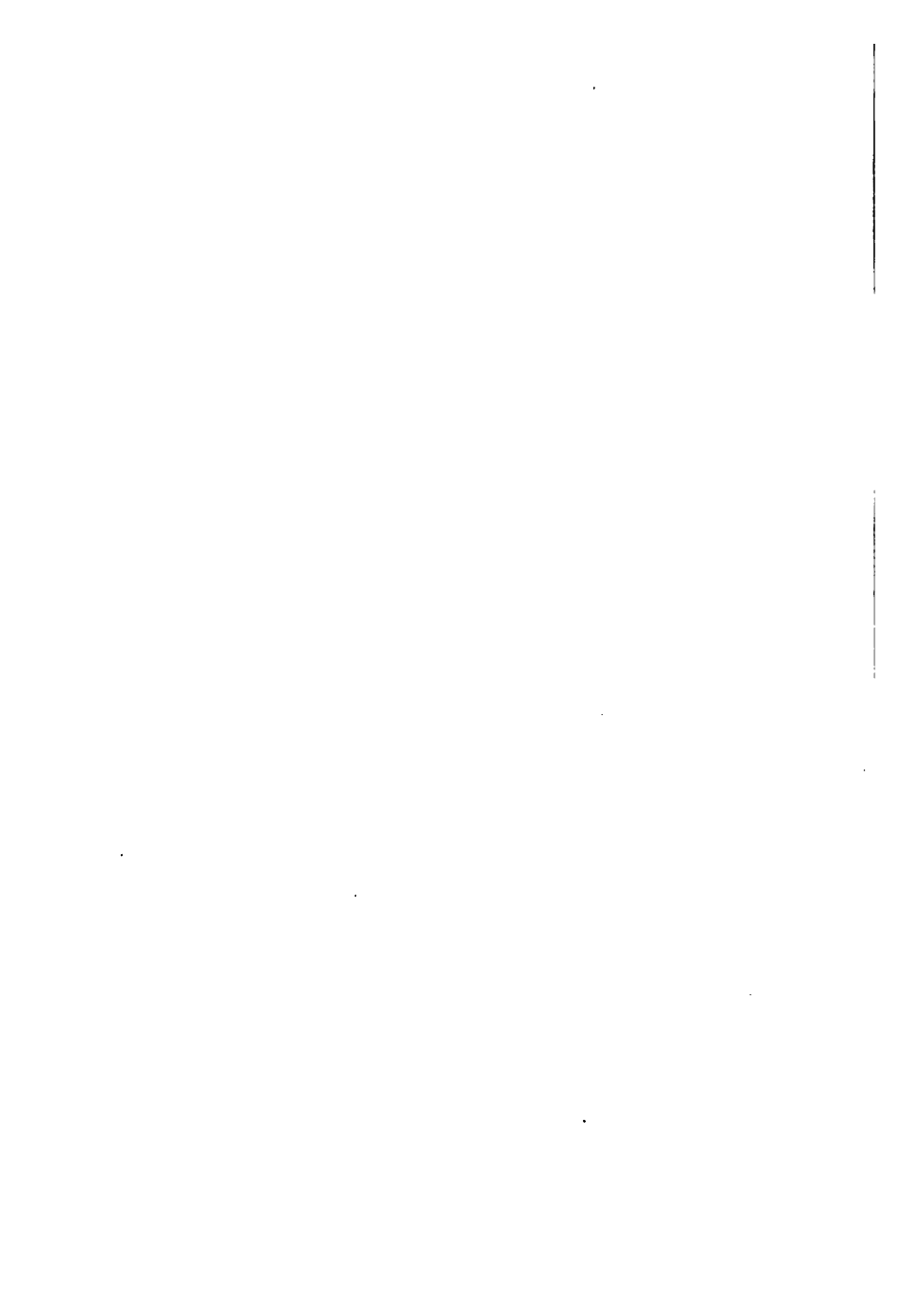
until that of force has been given, which, in his opinion, is a more fundamental concept than that of mass. The system of electromagnetic terms may be built up, starting either with electrostatic quantities or with magnetostatic concepts. Both methods are in use. It was therefore thought best, in defining such quantities as current, resistance and potential difference, to present both forms of definition.

The scope of a book of this kind is determined by the purpose for which it has been written. Since the present volume is intended to be used in connection with a first course of college or university physics and the earlier laboratory courses, no definitions of quantities which are mentioned only in more advanced courses are included. It was also thought best to omit definitions of physical instruments.

It is inevitable that much will be found open to criticism, and doubtless many of the definitions may still be improved. Any help in pointing out defects and errors will be highly appreciated. It is hoped that in spite of such defects the "Definitions" may be of as much help to the earnest student or teacher of elementary physics as the former edition has proved to be.

K. E. GUTHE.

UNIVERSITY OF MICHIGAN,
October, 1913.



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DEFINITIONS IN PHYSICS

FUNDAMENTAL DEFINITIONS

(a) *Physics* is that branch of the natural sciences which treats of the *general* properties of bodies in the outer world and of the general laws governing natural phenomena.

(b) *Fundamental Concepts*. — Certain concepts used in physics are deductions and generalizations from individual experience and cannot be strictly defined. Such are the concepts of *extension* (space, with its subdivisions of volume, area, length and direction), *time*, *force*, *warmth*, *cold*, etc.

The concepts of extension are closely related to such fundamental geometrical concepts as a point, line, surface, opening between two lines or surfaces, etc.

(c) A *physical quantity* is a definite concept, capable of measurement. It is either a fundamental concept or derived from fundamental concepts.

The use of a logically developed system of physical quantities greatly facilitates the description and classification of physical phenomena.

(d) A *physical law* is an equation between equivalent physical quantities studied from different points of view.

A physical law frequently expresses merely a proportionality between different physical quantities. In this case the proportionality factor forms a new physical concept which is often best defined by the equation. The equation always furnishes a means for the measurement of this new physical quantity.

(e) *Fundamental Assumptions* (Axioms).—(1) All physical phenomena take place, and all bodies are located, in a three-dimensional space; (2) all natural phenomena require time for their occurrence.

1. *Fundamental units* are the units of arbitrarily chosen physical quantities in terms of which the units of all other quantities may be expressed. The latter are called *derived units*.

Thus all units in mechanics are expressed in terms of those of mass, length and time. Other fundamental units are the degree of temperature (60), unit permeability (76) and unit dielectric constant (85).

2. The *unit of mass* in the metric system is the $\frac{1}{1000}$ th part of the mass of a standard, made of platinum-iridium and kept at the international Bureau of Weights and Measures at Sèvres, France. The unit is called a *gram* (g); one milligram (mg) = 0.001 g; one kilogram (kg) = 1000 g; one metric ton = 1000 kg.

The *unit of mass* in the English system is the mass of a platinum standard and is called a *pound*. One pound = 453.5924 g. In engineering a unit of mass is sometimes used which is 32.2 times as large as the mass of one pound. The names proposed for this unit, the *slug* or the *mydron*, have not been generally accepted. For a comparison of the different systems of units, derived from the different units of mass, see Table I, art. 36.

The standards are called "the international kilogram" and the "imperial pound avoirdupois." Two copies of the international kilogram are kept at the Bureau of Standards in Washington, D.C. They are called the American prototypes.

✓3. The *unit of length* in the metric system is the $\frac{1}{100}$ th part of the distance, at the temperature of melting ice, between two certain lines on the surface of a platinum-iridium bar preserved at the international Bureau of Weights and Measures at Sèvres, France. The unit is called a *centimeter* (cm). One meter (m) = 100 cm, one kilometer (km) = 1000 m, one millimeter (mm) = 0.1 cm, one *micron* (μ) = 0.001 mm.

Two American copies of the meter (American prototypes) are kept at the Bureau of Standards in Washington, D.C.

The unit of length in the so-called English system is the *foot* and is equal to one third of a *yard*. The United States yard is $\frac{3,600}{3,937}$ m.

One foot = 12 inches. One mile = 5,280 feet.
 One inch = 2.54001 cm. One foot = 0.304801 m.
 One mile = 1.6093 km.

Strictly speaking, the United States yard (= 91.4402 cm) differs in length from the British yard (= 91.4399 cm).

4. The *unit of time* in Physics is the $\frac{1}{86,400}$ th part of a *mean* solar day. It is called a *second*.

60 seconds = one minute.

60 minutes = one hour.

24 hours = one day.

One sidereal day = 23 hr 56 min 4.09 sec.

One solar year = 365 d 5 hr 48 min 46 sec.

One sidereal year = 365 d 6 hr 9 min 9 sec.

5. The *dimensional formula* of a physical quantity is an expression showing which of the fundamental units enter into its own unit, and their dimensions, as given by the indices with which they appear.

The symbols of the quantities used are inclosed in brackets. Thus the dimensional expression for mass is $[M]$; for length, $[L]$; for time, $[T]$. An example for the dimensional formula of a derived quantity (art. 28) is

$$[F] = [MLT^{-2}].$$

6. The *units of area*, A , and *volume*, V or τ , are generally derived from the units of length; thus a *square centimeter* (cm^2), a cubic centimeter (cc), a square foot and a cubic foot. One *are* = 10^6 cm^2 ;

one *hectare* = 100 ares. The *circular mil* is the area of a circle, one mil (= 0.001 in) in diameter. An *acre* = 4,840 square yards = about 0.4 hectare.

For liquid measures independent units are used as a rule. The unit of volume in the metric system is the volume of one kilogram of water at 4°C under atmospheric pressure. It is called the *liter*, and equals 1,000.028 cc. One liter = 1,000 milliliters.

In the English system one *gallon* = 231 cu in = 3.785 liters. The United States unit for dry measure is the *bushel* = 2,150.42 cu in.

The British units differ considerably from the American; thus one British gallon = 277.41 cu in and the British bushel = 2,219.28 cu in.

$$[\text{area}] = [L^2]; [\text{volume}] = [L^3].$$

✓ 7. The *density*, d , of a substance is its mass per unit volume.

$$d = m/V.$$

It is numerically equal to the mass in unit volume.

$$[d] = [ML^{-3}].$$

— *Unit density* is the *gram per cc*.

— The *specific gravity* of a substance is the ratio of its density to the density of water. In the case of gases air is often used as the standard.

It is a pure number; in the metric system the specific gravity of a substance is numerically equal to its density. This is exact only if the milliliter is chosen as the unit of volume.

— The *specific volume*, v , of a substance is its volume per unit mass.

$$v = \frac{V}{m}.$$

$$[v] = [L^3 M^{-1}].$$

(f) The *position of a body* with reference to another is determined by length and direction. A *direction* is determined by an angle.

8. An *angle*, θ , is the opening included between two intersecting straight lines. It is measured by the ratio of the subtended arc, s , of a circle, about the point of intersection as center, to its radius, R .

$$\theta = s/R.$$

Unit angle in the circular measure is the angle whose arc equals the radius; it is called a *radian*. Unit angle in the *degree* system is the 360th part of 2π radians. One radian = 57.2958 degrees.

$$1^\circ (\text{degree}) = 60' (\text{minutes}); 1' = 60'' (\text{seconds}).$$

$$[\theta] = [L]/[L]. \quad \text{An angle has no dimensions.}$$

A *solid angle*, ω , subtended by a surface at a point, lying outside the surface, is the opening formed by the bundle of straight lines, drawn from the given

point to every point of the boundary of the surface. A solid angle is measured by the ratio of the area A , cut out by the bundle of straight lines from a sphere about the given point as center, to the square of the radius R of the sphere.

$$\omega = \frac{A}{R^2}.$$

The solid angle subtended by a closed surface surrounding a point is 4π .

Unit solid angle is a solid angle cutting out unit area from the surface of a sphere of unit radius.

$[\omega] = [L^2]/[L^2]$. A solid angle has no dimensions.

I. MECHANICS

(*g*) *Mechanics* is that part of physics which treats of the motions and equilibrium of bodies and the *general* laws relating to them.

A. KINEMATICS

(*h*) *Kinematics* is that part of mechanics which treats of the motions of bodies without reference to the nature of the bodies or to the causes producing the motions.

9. A *material particle* is a geometrical point endowed with mass.

—10. A *scalar quantity* is a quantity having only magnitude, but no direction.

Some scalar quantities, as mass, volume and density are always positive; others, as temperature, potential and quantity of electricity, may be positive or negative.

—A *vector quantity* is a quantity having magnitude, direction and sense.

It may be positive or negative, according to its sense.

—A *vector* is a straight line representing a vector quantity.

The length of the line represents the magnitude, its orientation in the diagram the direction and an arrowhead the sense of the vector quantity.

A *rotor* is a vector which must be drawn through a definite point. It is a localized vector.

11. The *distance* between two points is the length of the straight line connecting them.

The *distance from* one point to another is a vector quantity.

12. A *displacement* is a change of position.

It is a vector quantity and may be measured, either in linear or angular units, or both.

An angular displacement is represented by a vector, perpendicular to the plane of the angle, and of length, numerically equal to the angle. The sense in which a right-handed screw will advance, if given a twist, is taken as the positive sense of the angular displacement of the screw.

—13. *Motion* is the (continuous) process of change of position.

a. *Rectilinear motion* is motion in an unvarying direction.

b. *Curvilinear motion* is motion with varying direction.

— *Curvature*, C , is the rate of change of direction with respect to the distance, or the *space rate* of change of direction.

Change of direction of a curve is measured by the angle through which the tangent to the curve turns.

$$C = \frac{d\theta}{ds} = \frac{1}{R};$$

$$[C] = [L^{-1}].$$

The average rate of change of a physical quantity, y , with respect to another, x , is the ratio of the difference in value of the first quantity to the corresponding change of the other. The former quantity is called the dependent, the second the independent variable. In general, this ratio is not a constant. In order to render the expression exact for a given point or instant the change of the independent variable must be taken very small, or, in the language of calculus, the ratio $z = \frac{\Delta y}{\Delta x}$ must

be found for the limit $\Delta x = 0$, which is expressed by $z = \frac{dy}{dx}$; for a finite change of y we have $y_1 - y_2 = \int_{x_1}^{x_2} z dx$. The average value of the ratio in the notation of calculus is: $z = \frac{\int z dx}{\int dx}$.

14. Translation of a body is a motion in which the displacements of all points of the body are linear, parallel and equal.

The motion of a body in translation is completely defined when the motion of one point of the body is given.

Rotation of a body is a motion in which the displacements of all points of the body are angular and equal.

The *axis of rotation* is the straight line formed by the centers of the circular paths described by the points of the rotating body.

The motion of a rotating body is completely defined by the direction of the axis of rotation and the motion of any one point of the body.

- 15. *Velocity* at a given instant is the time rate of motion. ✕

It may be either variable or constant. It is a vector quantity.

a. *Linear*, or rectilinear, *velocity*, v , is the time rate of motion resulting in rectilinear displacement.

$$v = \frac{ds}{dt}, \text{ and } s = \int v dt.$$

It is numerically equal to the distance passed over in the next second if the velocity remains constant.

$$[v] = [LT^{-1}].$$

- Its *unit* is a *velocity of one centimeter per second*, or simply, the *centimeter per second* (metric system), or *one foot per second* (English system).

The *average linear velocity* during the time, t , is :

$$v = \frac{s}{t}.$$

- b. *Angular velocity*, ω , is the time rate of motion ✕
resulting in angular displacement.

$$\omega = \frac{d\theta}{dt}; \quad \theta = \int \omega dt.$$

It is a vector quantity and represented similarly to angular displacement (art. 12).

$$[\omega] = [T^{-1}].$$

Its *unit* is an *angular velocity of one radian per second*, or of one degree per second.

- × *c. Speed, \bar{v} , is the time rate with which a body moves along its path, without reference to direction.*

$$\bar{v} = \frac{ds}{dt}; \text{ average speed, } \bar{v} = \frac{s}{t}.$$

It is the magnitude of the velocity.

— Its *unit* is one centimeter per second.

16. *Uniform rectilinear motion* is a motion of constant linear velocity.

$$s = vt + s_0.$$

- × *Uniform angular motion* is a motion of constant angular velocity.

$$\theta = \omega t + \theta_0.$$

If a material particle moves uniformly along a circular path, this motion is called *uniform circular motion* (art. 19).

17. *Accelerated motion* is a motion of variable velocity.

— *Acceleration* is the time rate of change of velocity.

It is a vector quantity and may be either variable or constant.

a. Linear acceleration, a , is the time rate of change of linear velocity.

$$a = \frac{dv}{dt}.$$

It does not need to be in the same direction as the motion of the body.

$$[a] = [LT^{-2}].$$

Unit linear acceleration is the change of velocity by one unit (for example: one centimeter per second) in one second. In the metric system it is the *centimeter per second per second*; in the English system the *foot per second per second*.

Gravity, or acceleration due to gravity, g , is the linear acceleration due to the earth's attraction.

Its value (g) found at a place, H meters above sea level, may be reduced to sea level by means of the equation

$$g_0 = g + 0.0003086 H.$$

Helmert's equation for g at sea level and geographical latitude ϕ :

$$g_0 = 978.03 (1 + 0.005302 \sin \phi - 0.000007 \sin^2 \phi).$$

— *b. Angular acceleration*, α , is the time rate of change of angular velocity.

$$\alpha = \frac{d\omega}{dt}.$$

$$[\alpha] = [T^{-2}].$$

Unit angular acceleration is an acceleration of one radian, or one degree, per second per second.

— 18. *Uniformly accelerated motion* is a motion with constant acceleration. If the acceleration is in the direction of the motion

$$a = \frac{d^2s}{dt^2}; s = \frac{1}{2}at^2 + v_0t; v = at + v_0.$$

19. *Uniform circular motion* of a particle is a motion of constant speed in a circular path.

$$s = \bar{v}t; \quad a = \frac{\bar{v}^2}{r}.$$

The linear acceleration, a , while constant in magnitude and sense changes its direction with constant angular velocity. It is at right angles to the path.

$$\bar{v} = \omega r; \quad a = \omega^2 r$$

In general the acceleration of a particle in curvilinear motion may be resolved into two components, (1) tangential to the path and equal to $\frac{d^2s}{dt^2}$; (2) at right angles to the path and equal to $\frac{v^2}{r}$, where r is the radius of curvature of the path at the point under consideration.

The *period* T of a uniform circular motion is the time in which the particle passes once around the circle, or, the time interval between two successive passages of the particle through the same point in the same sense.

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}.$$

$$[\text{Period}] = [T].$$

X 20. *Simple harmonic motion* is rectilinear motion in which the (linear) acceleration is directed towards a center and is proportional to the distance of the moving particle from it.

$$\frac{d^2x}{dt^2} = a = -cx; \quad v = \pm \sqrt{c} \sqrt{r^2 - x^2}.$$

— The *amplitude* r of simple harmonic motion is the greatest distance from the center reached by the vibrating particle.

Circle of reference is a circle constructed around the center of simple harmonic motion, one of the diameters being the path of the vibrating particle.

Its radius is therefore the amplitude of the simple harmonic motion.

Reference point is a point assumed to move on the circle of reference with uniform circular motion so that its projection upon the diameter coincides at every instant with the particle in simple harmonic motion.

— *Period of vibration* is the time interval between two successive passages of the particle through the same point in the same sense.

$$T = 2\pi\sqrt{-\frac{x}{a}}.$$

In simple harmonic motion the period is often used as the unit of time.

$$[\text{Period}] = [T].$$

Frequency is the time rate of vibration : $n = \frac{1}{\text{period}}.$

It is numerically equal to the number of vibrations made in unit time.

$$[n] = [T^{-1}].$$

- *Phase* is the fraction of a whole period which has elapsed since the body last passed through the center in the positive sense.
- *Phase angle*, ϕ , is the angle swept out by the reference point, since it last passed through the point corresponding to the center of simple harmonic motion.

It is frequently used to measure the phase: $\text{phase} = \frac{\phi}{2\pi}$.

Time angle, θ , is the angle swept out by the reference point since zero time, *i.e.*, since time was begun to be counted.

$$\theta = \omega t.$$

In plotting the position of a particle in simple harmonic motion at different times the time angle is frequently used instead of the time.

Epoch angle, ϵ , is the phase angle at zero time.

$$\epsilon = \phi - \omega t.$$

$$x = r \sin \phi = r \sin(\omega t + \epsilon); \quad a = -\omega^2 r \sin \phi = -\omega^2 x.$$

21. *Angular harmonic motion* about an axis is an angular motion in which the angular acceleration is opposite in sense and proportional to the angular displacement.

$$\alpha = -C\theta; \quad T = 2\pi\sqrt{-\frac{\theta}{\alpha}}.$$

22. *Damped harmonic motion* is harmonic motion with decreasing amplitude (such that the ratio of successive amplitudes remains constant).

$$\frac{d^2x}{dt^2} + 2a\frac{dx}{dt} + b^2x = 0; \quad x = Xe^{-at} \sin \omega t; \quad T = \frac{2\pi}{\sqrt{b^2 - a^2}}.$$

Logarithmic decrement is the natural logarithm of the ratio of two successive amplitudes.

$$\lambda = aT.$$

23. *Pendulum motion* is a motion under the influence of a constant acceleration, g , the particle being constraint to move in a circle, of radius l , whose plane is parallel to the direction of the acceleration.

$$T = 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{1}{4}\sin^2\frac{\alpha}{2} + \frac{9}{64}\sin^4\frac{\alpha}{2} + \dots\right).$$

24. *Wave motion* is a mode of transferring a disturbance through an elastic medium accompanied by periodic variations in the physical condition of the portions of the medium involved.

It is characterized by uniform rectilinear motion (art. 16).

$$s = vt.$$

Waves are moving geometrical figures formed by vibrating particles.

They may exist in either one, two or three dimensional space.

We speak of the motion and the velocity of a wave in the same way as of the motion and velocity of a material particle.

The term "*wave*" is generalized so as to be applicable to any form of wave motion, even though no definite geometrical figure of the wave exists. We speak thus of light waves, temperature waves, etc.

A *simple wave* is a wave represented by a sine curve.

The displacements of a particle in simple harmonic motion (art. 20) if plotted as a function of the time, form a simple wave.

A *wave front* is a continuous surface containing points of the wave that are in the same phase of vibration.

A *wave length* is the distance between a wave front and the next wave front having points in the same phase. It is the distance traversed by a wave during one period.

$$\lambda = v/n$$

where v is the velocity of the wave and n the frequency of vibration.

The second definition allows the measurement of wavelength, if the term "*wave*" is used in its generalized meaning.

$$[\lambda] = [L].$$

Amplitude of a wave is the maximum variation at a given place from the condition of rest.

If the wave motion is due to simple harmonic motion of particles, the amplitude of the wave becomes identical with that of the simple harmonic motion.

25. A *longitudinal vibration* is a vibration in which the particles vibrate in the direction in which the wave is propagated, or, in general, in which the periodic disturbance is that of a vector lying in the direction of the propagation of the wave.

A *transverse vibration* is a vibration in which the particles vibrate at right angles to the direction in which the wave is propagated, or, in general, in which the varying vector is at right angles to the direction of the propagation of the wave.

A *torsional vibration* is a vibration in which the particles vibrate in circles whose planes are perpendicular to the direction of propagation of the wave.

26. *Interference* is the destructive or reënforcing action of different systems of waves upon each other.

Diffraction is the bending of the line of propagation of wave motion around the edge of an obstacle.

It is an interference phenomenon.

B. DYNAMICS

(i) *Dynamics* is that part of mechanics which treats of the forces acting upon bodies and of the motions or equilibria produced by forces.

(j) *Axiom 3.* — All bodies relatively at rest remain at rest, and bodies in motion continue in motion, which will be uniform rectilinear motion, unless disturbed by outside influences.

- × 27. *Inertia* of a body is its persistence in its condition of rest or uniform rectilinear motion. It is measured by the mass of the body.
- × 28. *Force* is the cause of a change or of a tendency to change in the state of rest of a body or of its deviation from uniform rectilinear motion.

The idea of force is based upon the fundamental concept of the effort necessary to change the position or the uniform rectilinear motion of a body. It is assumed to be the cause of such a change, to be proportional to the acceleration produced, and to be in the direction of the acceleration. It is a vector quantity.

A force is measured by the equation

$$F = ma,$$

which may serve as a definition.

$$[F] = [MLT^{-2}].$$

- *Unit force* (absolute system) is that force which gives unit mass unit acceleration. In the metric system it is the force which gives one gram an acceleration of one centimeter per second per second. It is called a *dyne*. In the English system unit force is called a *poundal*.
- The *weight*, w , of a body is the force of attraction of the earth for the body, resisting an effort to lift it.

$$w = mg.$$

This gives a new definition of mass: The *mass* of a

body is a characteristic property of the body, proportional, at a given place, to its weight.

It is commonly measured by a comparison of the weight of the body with that of a standard (art. 2).

— *Unit weight* (gravitational system) is the force which gives unit mass an acceleration equal to gravity. In the metric system it is called a *gram weight*; in the English system unit weight is called a *pound weight*, or simply, a *pound*.

One gram weight = 981 dynes, nearly.

If a force is applied to a body and no change of motion results from it (i.e., if the body is in equilibrium with respect to the force), a reacting force or resistance equal to the applied force and in the opposite direction is produced, accompanied by some change in the interacting bodies; for example, compression or elongation.

— 29. *Momentum*, or quantity of motion, N , of a moving body, is the product of its mass and velocity:
 $N = mv$.

$$[N] = [MLT^{-1}].$$

It is a vector quantity.

The *unit of momentum* is the momentum of a gram moving with a velocity of one centimeter per second. The name *bole* has been proposed for this unit.

30. *Impulse*, I , is the effect of a force upon the state of motion of a body. It is measured by the product of the force and the time during which it acts.

$$I = \int_0^t F dt = m(v_t - v_o).$$

It equals the change of momentum produced.

$I = Ft$, where F is the average force during the time t .

31. A *gravitational force* is a force measuring the attraction between two material particles, m_1 and m_2 , a distance, r , apart, according to the law of gravitation.

$$f = G \frac{m_1 m_2}{r^2}.$$

The *constant of gravitation* is the factor G in the law of gravitation, needed to reduce the force to common units. In the metric system its numerical value is $G = 0.0000006658$.

32. A *centripetal force* is a force changing the direction of motion of a material particle, normal to the path of the particle.

It is constant in magnitude for a mass moving in uniform circular motion.

$$F_n = m \frac{v^2}{r}.$$

33. A *frictional force* is a force resisting the free motion of a body under the influence of other forces. The term is usually applied to the gliding motion of one surface upon another.

The *coefficient of static friction*, μ , is the ratio of a force, F , just able to move a body along a surface, to the force F' exerted by the body normally to the surface.

$$\mu = \frac{F}{F'}.$$

The *coefficient of kinetic friction*, μ' , is the ratio of the force, F_1 , able to maintain uniform motion of a body along a surface, to the force F_1' , exerted by the body normally to the surface.

$$\mu' = \frac{F_1}{F_1'}.$$

— 34. *Pressure*, at a point, is the force per unit area pressed upon.

$$P = \frac{dF}{dA}.$$

— The *unit of pressure* is the dyne per square centimeter. One *atmosphere* = 1,012,630 dynes per cm^2 . Similar definition for *tension*.

— 35. *Work* is the product of a force and the displacement produced by or against this force in the direction of the force

$$W = \int F ds \cos \alpha,$$

where α is the angle between the direction of the force and the displacement.

If F is constant, $W = Fs \cos \alpha$.

$$[W] = [ML^2T^{-2}].$$

Though work is the product of two vector quantities, it is a scalar quantity. In general, the product of two vector quantities parallel to each other is called the scalar product or dot product of the two vector quantities. If work is done upon a system, it is usually considered positive; if done by the system, negative.

Unit work, in the absolute system, is the work done if unit constant force produces unit displacement in the direction of the force. In the metric system it is the work done by a dyne in producing a displacement of one centimeter. It is called an *erg*. One *joule* = 10^7 ergs. In the English system the unit is called a *foot poundal*.

Unit work, in the gravitational system, is the work done if unit weight produces unit displacement in the direction of the force. In the metric system it is the gram-weight-centimeter. The *kilogrammeter* = 100,000 such units.

In the English system the unit is called a *foot pound*.

36. *Power* is the time rate of doing work.

$$\mathcal{P} = \frac{dW}{dt}.$$

It is numerically equal to the work done in unit time, if the power is constant.

$$[\mathcal{P}] = [ML^2T^{-3}].$$

Unit power is the constant power by which unit work is done in unit time.

In the absolute metric system it is the *erg-per-second*. The *watt* equals 10^7 such units, or one joule per second. In the gravitational metric system it is the *kilogrammeter-per-second* and in the English gravitational system the *foot-pound-per-second*. A *horse power* equals 550 foot-pounds-per-second = 0.7456 kilowatt = 745.6 watts.

A *metric horse power* = 736 watts = 75 kilogrammeter-per-second.

A unit of work, frequently used, is the *kilowatt hour*. One kilowatt hour = 3,600,000 joules.

TABLE I

METRIC SYSTEM			ENGLISH SYSTEM	
	Absolute	Gravitational	Absolute	Gravitational
Force	Dyne	Gram weight	Poundal	Pound
Work	Erg, joule Watt hour	Kilogrammeter	Foot poundal	Foot pound
Power	Watt			Horse power

37. The *energy*, W , of a body or system of bodies is a physical quantity of the nature of work by virtue of which the body or system of bodies is able to produce external effects. It is capacity for doing work and may be measured by the work necessary to change

the state of the body or system of bodies to another state selected as representing zero energy for the given problem.

It is therefore always relative. The energy of a system decreases if the work is done by internal forces; it increases if the work is done against these forces.

Energy may appear in many different aspects, according as we are considering phenomena in mechanics, heat, electricity, etc., and it is in these cases usually expressed in terms of different units. It has always the dimensions of work.

Energy is a physical quantity which remains constant in a given isolated system irrespective of any changes which may occur in the system.

$$[W] = [ML^2T^{-2}].$$

The *unit of energy*, when measured in mechanical units, is the unit of work (see 35).

— *Potential energy* is energy of position. It is measured by the work required to produce the given configuration.

$$\text{Potential energy} = \int F \cos \alpha \, ds.$$

— *Kinetic energy* is energy of motion. It is expressed in terms of the mass of the body and its velocity.

$$\text{Kinetic energy} = \frac{1}{2} mv^2.$$

It is equal to the work required to produce the momentum, mv .

$$\text{K. E.} = \int F ds = \int_0^v mvdv = \frac{1}{2} mv^2.$$

38. A *conservative force* is a force whose action leaves the total mechanical energy (potential and kinetic) of a system of bodies unchanged. The work done in bringing a system which is under the action of conservative forces from one configuration to another is independent of the manner in which the change of configuration takes place.

Gravitational or elastic forces are conservative forces.

A *nonconservative* or *dissipative force* is a force under whose action the mechanical energy of a system changes.

Frictional forces are nonconservative forces.

— 39. The *torque* or *moment of a force* about a straight line is the cause of rotation or tendency to produce rotation about this line as axis. It is measured by the product of the perpendicular distance, l , from the axis to the line of action of the force and the component, F , of the force normal to the axis.

$$\mathcal{J} = lF = rF \sin \alpha.$$

It is a vector quantity.

A torque may also be referred to a point as center of rotation.

$$[\mathcal{J}] = [ML^2T^{-2}].$$

40. The *moment of a mass* of a material particle with respect to a point, line or plane is the product

of the mass into its distance from the point, line or plane.

- *Center of inertia, center of mass or centroid* of a system of particles or of a body is a point so situated that a translation of the body would not be changed if the total mass were concentrated at this point and the resultant of all parallel forces acting on the separate particles were applied at this point.

It is located by making use of the fact that its moment with respect to any plane is equal to that of the whole system with respect to the same plane.

$$\bar{x} = \frac{\sum mx}{M}; \quad \bar{y} = \frac{\sum my}{M}; \quad \bar{z} = \frac{\sum mz}{M}.$$

This property may also be used as the definition of the center of inertia.

Center of gravity of a body is a point so situated that if the total mass were concentrated at this point, the translation of the body due to its weight would not be changed.

The center of gravity of a small body coincides with its center of inertia, since the forces acting on the different particles are parallel.

- 41. The *moment of inertia, I*, of a system of particles or of a body with respect to a line is its persistence to remain in uniform rotation about this line, unless acted upon by a torque. It is the proportionality factor between torque and angular acceleration.

$$\alpha I = \mathcal{T}.$$

It is measured by the sum of the products of the mass of each particle and the square of its distance from the line.

$$I = \sum mr^2.$$

Moment of inertia may also be referred to a point or a plane.

In engineering practice the sum of the products of each elementary area and the square of its distance from a point or line is, because of its mathematical similarity to the above expression, called the moment of inertia of an area.

42. *Angular momentum* or *moment of momentum* of a body about an axis is the product of the moment of inertia into the angular velocity about this axis.

— *Radius of gyration* is the distance from the axis of rotation, of a point where the mass of the body may be assumed to be concentrated without changing the *moment* of inertia.

(k) A moment of a force produces rotation. The mathematical equations for rotation take the same form as those of translation if the angular units are substituted for the corresponding linear ones.

TABLE II

TRANSLATION	ROTATION	TRANSLATION	ROTATION
l	θ	$l = vt + \frac{1}{2}at^2$	$\theta = \omega t + \frac{1}{2}\alpha t^2$
v	ω	$F = ma$	$\mathcal{J} = I\alpha$
a	α	$W = Fl$	$W = \mathcal{J}\theta$
F	\mathcal{J}	$\mathcal{J} = Fv$	$\mathcal{J} = \mathcal{J}\omega$
m	I	$\text{Kin. En.} = \frac{1}{2}mv^2$	$\text{Kin. En.} = \frac{1}{2}I\omega^2$

- 43. *Elasticity* is a property of bodies shown by a tendency to recover from a deformation. If the deformation consists of a change of volume, we speak of *volume elasticity*; if it is one of shape only (a shear), we speak of the *rigidity* of a body.

An *elastic force* is a force due to an elastic deformation. It is proportional to the deformation and in the opposite direction; for example, for a stretched wire or spring

$$F = - Cx.$$

- 44. *Strain* is any change of volume or shape or of both. It is measured by the relative deformation; for example, strain = $\Delta l/l$.

Strain has no dimensions.

A *shearing strain*, or *shear*, is a change of shape only and it is measured by the angle (or tangent of the angle) through which a straight line, originally normal to the distorting force, has been displaced.

- A *stress* is the action and reaction of internal forces. It is measured by the force per unit area tending to restore the body to its original shape or volume.

$$[\text{stress}] = [ML^{-1}T^{-2}].$$

- 45. *Modulus*, or *coefficient of elasticity*, is the quotient of stress divided by the strain.

After equilibrium has been established between the distorting and resisting forces, the stress may be taken as equal to the distorting force (WORKING STRESS) divided by the area upon which it acts.

Modulus of volume elasticity

$$= \frac{dF}{A} : \frac{dV}{V} = \frac{V}{A} \frac{dF}{dV} = \frac{dP}{dV} V.$$

Young's modulus = $dF/A + dl/l$

$$= dFl/Adl = \frac{l}{A} \frac{dF}{dl}.$$

Modulus of rigidity = $\frac{\text{shearing stress}}{\text{shearing strain}}$

$$= \frac{dF}{A} : d\theta = \frac{1}{A} \frac{dF}{d\theta}.$$

[Modulus] = $[ML^{-1}T^{-2}]$. Unit: The dyne per square centimeter.

46. *Elastic aftereffect* is the gradual increase of a strain before equilibrium is established under a constant distorting force.

The *energy* stored in a strained elastic body (potential energy) is measured by the work done in producing the strain; for example, in a stretched wire or spring

$$W = \int F dx = - \int_{l_1}^{l_2} - Cx dx = \frac{1}{2} C(l_2^2 - l_1^2).$$

The *coefficient of restitution*, e , between two colliding bodies is the ratio of the speed of separation after impact to the speed of approach before impact.

$$e = \frac{v_1 - v_2}{v_2' - v_1'}$$

47. A *homogeneous body* is a body which has at all points the same properties.

An *isotropic body* is a body which at any point has the same properties in all directions.

— 48. A *solid* is a body which possesses both volume elasticity and rigidity.

A *crystalline solid* is a solid of definite geometrical form or composed of particles of definite structure.

An *amorphous solid* is a solid without any definite structure.

— 49. A *fluid* is a body which possesses only volume elasticity.

It yields to any continued shearing force, however small.

A *liquid* is a fluid capable of forming a free surface ; *e.g.*, the level surface of water in a beaker.

A *gas* is a fluid incapable of forming a free surface.

It will completely fill any vessel in which it is contained.

The division of bodies into solids, liquids and gases facilitates the treatment of problems in mechanics. It may lead to

inconsistencies in other branches of Physics. Thus an amorphous solid may well be considered as a hardened liquid (art. 65) and under certain conditions no distinction can be made between liquids and gases (art. 69).

50. *Viscosity* is the property of a fluid by virtue of which a shear can be established only in finite time.

The *coefficient of viscosity*, η , of a fluid is the ratio of the shearing stress in the fluid to the time rate of the shear

$$\eta = \frac{\frac{F}{A}}{\frac{\tan \theta}{t}} = \frac{\frac{F}{A}}{\frac{v}{d}},$$

where F is the force in the plane of the surface, A , producing in the time, t , an angular shear, θ , of a line, originally perpendicular to the surface and at a distance, d , from another surface at rest.

η becomes numerically equal to F if the other quantities in the expression become unity.

$$[\eta] = [ML^{-1}T^{-1}].$$

51. *Hydrostatic pressure* at a point in a fluid at rest is the pressure exerted by the fluid at the point. It is a stress. If the pressure is due to the weight of the fluid alone,

$$P = h\bar{d}g = \int_0^h \bar{d}g dx,$$

where h is the distance from the surface of the fluid, \bar{d} its density and g the acceleration due to gravity.

Coefficient of compressibility, α , is the ratio of the relative change of volume to the pressure producing it, the temperature remaining constant.

$$\alpha = \frac{1}{V} \frac{dV}{dP}.$$

It is the reciprocal of the modulus of volume elasticity.

A *thrust* (or total pressure) is the force exerted by a fluid upon a surface.

$$F = \int P dA.$$

If the pressure is uniform over the surface,

$$F = PA.$$

It is always at right angles to the surface.

The *center of hydrostatic pressure* on a surface, immersed in a fluid, is the point of application of the resultant of all forces on the surface due to the fluid, or, the point of application of the thrust.

It is found by the condition that the sum of the moments of all the elementary forces about this point must be zero.

$$\bar{x} = \frac{\iint x P dx dy}{\iint P dx dy}; \quad \bar{y} = \frac{\iint y P dx dy}{\iint P dx dy}.$$

Work due to the expansion of a fluid is measured in terms of the pressure and change of volume.

$$W = \int F ds = \int P dV.$$

If the pressure is uniform,

$$W = P(V_2 - V_1).$$

52. The *buoyant force* or *buoyancy* of a fluid on a body immersed in it is the resultant upward force due to the pressure of the fluid on the surface of the body. It is measured by the loss of weight of the body when it is immersed in the fluid.

$$F = W - w' = Vdg,$$

where d is the density of the fluid. It is equal to the weight of the fluid displaced.

The *center of buoyancy* is the point at which the buoyant force may be assumed to act. It is the center of inertia of the displaced fluid.

53. *Surface tension* of a liquid is the ratio of the force exerted by the surface of the liquid to the length of the boundary line.

$$T = \frac{F}{l}.$$

The force is always at right angles to the boundary and parallel to the surface of the liquid.

$$[T] = [MT^{-2}].$$

II. SOUND

(1) *Sound*, or *Acoustics*, is that part of Physics which treats of the physical phenomena which may affect the sense of hearing, and of the laws governing these and other phenomena of the same physical nature.

Sound as recognized by the ear is due to a vibratory motion in an elastic body, and is propagated through air as wave motion.

54. *Pitch* is the factor of sound sensation depending on the frequency of the vibrations producing the sound. As a physical quantity it is the frequency.

$$n = \frac{v}{\lambda}.$$

$$[n] = [T^{-1}].$$

55. The *fundamental* of a vibrating body or of a sound is the component vibration of greatest wave length (lowest pitch).

Overtones are waves of shorter wave length than the fundamental.

Harmonics are overtones whose frequencies are exact multiples of the fundamental.

The wave form of a given sound depends upon the number of overtones combined with the fundamental, the amplitude of the different components and their difference in phase.

The *quality of a sound* is the factor of sound sensation depending upon the manner in which the fundamental is combined with the overtones. As a physical quantity it is the wave form.

56. *Intensity* of a sound at a point is the time rate of transmission of energy per unit area at the point.

It varies as the square of the amplitude of the wave.

The *loudness* of a sound is the factor of sound sensation depending upon the intensity and upon the sensitiveness of the ear.

Beats are periodic variations in the intensity of a wave due to the interference of two or more components of different wave lengths.

57. *Stationary wave motion* is a form of vibration characterized by the existence of certain points, lines or surfaces at rest.

Nodes are the points, lines or surfaces of a body in stationary wave motion which remain at rest.

They are characterized by maximum variation of stress (pressure in the case of gases).

Antinodes are the points, lines or surfaces of a body in stationary wave motion which have a maximum amplitude.

They are characterized by maximum variation of the kinetic energy of the moving parts.

All sources of sound are bodies in stationary vibration. The parts of the vibrating body are in simple harmonic motion, while the wave form does not progress along the body.

58. A *musical tone* is a train of sound waves of constant frequency.

A *musical note* is a tone whose pitch is referred to that of a standard. In Physics this standard is usually $n = 256$ per second and is called c^2 ; in music it is $n = 435$ per second and is called a^1 .

A *musical interval* is the ratio of the frequencies of two tones.

An *octave* is the interval between two tones whose frequencies are in the ratio of 2 to 1.

A *musical scale* is a series of notes ascending or descending in pitch by definite intervals.

A *pure musical scale* is a scale in which the intervals are simple fractions.

Intervals used in this scale are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$.

An *equally tempered scale* is a scale in which all intervals are equal.

The interval used in this scale is $\sqrt[12]{2} = 1.0594+$.

59. The *major triad* is a combination of three tones whose frequencies are as 4 : 5 : 6.

The *minor triad* is a combination of three tones whose frequencies are as 10 : 12 : 15.

We may also speak of a major or a minor triad in a tempered scale though the ratios of the frequencies are not exactly those given above.

The *diatonic scale* is a scale containing seven notes in an octave and based upon the use of either the major or the minor triad.

The *chromatic scale* is a scale of twelve notes, obtained from the diatonic scale by the addition of five semitones.

The complete series of tones on a piano form an equally tempered chromatic scale.

III. HEAT

(*m*) *Heat* is that part of Physics which treats of the physical phenomena affecting the sense of warmth and cold and of the laws governing these and other phenomena of the same physical nature.

60. *Temperature* is the condition of a body affecting the sensations of warmth and cold. Its changes are accompanied by certain physical changes; for example, changes in pressure, volume, electrical resistance, electromotive force, etc., any one of which may be chosen as a basis for measurement of temperature.

In the *mercury in glass thermometer* the increase of temperature is taken as proportional to the apparent expansion of mercury.

In the *standard hydrogen thermometer* the increase in temperature is taken as proportional to the increase in pressure of a constant volume of hydrogen gas originally under a pressure of 100 cm of mercury at the temperature of melting ice.

$$t_2 - t_1 = c(p_2 - p_1) = \frac{\alpha}{p_0}(p_2 - p_1).$$

For accurate measurements all thermometric readings must be reduced to the standard hydrogen thermometer.

A *temperature scale* is a series of temperatures, forming an arbitrary number of subdivisions of a

chosen temperature interval and counted from an arbitrary zero point, but subject to the selection of the particular type of thermometer.

The *fundamental interval* is the temperature interval selected for the construction of a temperature scale. It is the interval between the temperature of melting ice and that of steam of pure water boiling under the pressure of one atmosphere.

The *unit of the temperature scale* is called a *degree*. In the *Centigrade scale* it is the $\frac{1}{100}$ th part of the interval between the temperatures of melting ice and of the steam of pure water boiling under one atmosphere. The former point is chosen as zero. In the *Fahrenheit scale* it is the $\frac{1}{180}$ th part of the same interval and the melting point of ice is chosen as $+32$ degrees. In the *Réaumur scale* it is the $\frac{1}{80}$ th part of the same interval and the melting point of ice is chosen as zero.

Temperature is a fundamental unit and its dimensional expression is $[\theta]$.

61. *Temperature coefficient* is the ratio of the relative change of a physical property to the corresponding change of temperature, for example :

Coefficient of linear expansion (increase in length per unit length per degree),

$$\alpha = \frac{1}{L} \frac{dL}{dt}.$$

Coefficient of cubical expansion, or simply *coefficient of expansion* (the pressure remaining constant),

$$\alpha_p = \frac{1}{V} \frac{dV}{dt}.$$

Pressure coefficient (the volume remaining constant),

$$\alpha_v = \frac{1}{P} \frac{dP}{dt}.$$

$$[\alpha] = [\theta^{-1}].$$

Zero coefficient is the temperature coefficient referred to the condition at zero degree, for example :

$$\alpha_v = \frac{1}{P_0} \frac{dP}{dt}.$$

Since in the Centigrade scale of the hydrogen thermometer α_v is constant and P equals P_0 at zero degree :

$$P_t = P_0(1 + \alpha_v t),$$

and
$$t = 100 \frac{P_t - P_0}{P_{100} - P_0}.$$

For other physical quantities :

$$L_t = L_0(1 + \alpha t + \beta t^2 + \dots).$$

$$V_t = V_0(1 + \alpha_v t + \beta_v t^2 + \dots).$$

"Reduced to the hydrogen scale" means "reduced to the centigrade scale of the hydrogen thermometer."

62. *Absolute zero* on the gas scale is the temperature at which hydrogen gas would theoretically exert no pressure. It is numerically equal to the reciprocal of the pressure coefficient of the gas: $= 1/\alpha_p$.

Absolute zero $= -273.09^\circ \text{C}$.

Absolute temperature is the temperature reckoned from absolute zero.

$$T^\circ = \frac{1}{\alpha_p} + t^\circ.$$

63. *Heat* is a quantity which, when added to a body, produces changes in the physical condition of the body, accompanied, in general, by a rise of temperature. Heat has been shown to be equivalent to energy and may therefore also be defined as energy measured in terms of thermal units.

$$[H] = [ML^2T^{-2}].$$

The *unit of heat* is the heat necessary to raise the temperature of unit mass of water one degree. In the metric system it is called a *calorie*; in the English system a *British thermal unit* (B. T. U.).

The heat necessary to raise the temperature of a gram of water one degree Centigrade varies slightly with the temperature. The calorie, therefore, becomes a definite quantity only when the temperatures selected for its determination are specified. The interval from 15° to 16°C . is generally chosen.

64. The *thermal capacity of a body* is the ratio of the heat received by the body to the change of temperature produced :

$$C = \frac{H}{t_2 - t_1}, \text{ or } H = C(t_2 - t_1).$$

In the metric system it is numerically equal to the number of calories required to raise the temperature of the body one degree Centigrade.

$$[C] = [ML^2T^{-2}\theta^{-1}].$$

The *thermal capacity of a substance* is the ratio of the thermal capacity of a body to its mass :

$$c = C/m, \text{ or } H = cm(t_2 - t_1).$$

It is numerically equal to the thermal capacity of unit mass of the substance.

From the definition of the calorie it follows that the thermal capacity of water is unity. It varies slightly with the temperature.

$$[c] = [L^2T^{-2}\theta^{-1}].$$

The *specific heat* of a substance is the ratio between its thermal capacity and that of water :

$$s = c_x/c_w.$$

It is numerically equal to the thermal capacity of the substance. It has no dimensions.

65. *Fusion* is the transition from the solid to the liquid state. The *melting point* or *freezing point* of a

crystalline body is the temperature at which fusion or freezing takes place.

It is usually referred to one atmosphere.

The *fusion curve* is a curve on the temperature-pressure diagram representing the melting point as a function of the pressure.

The fusion curve represents the stable equilibrium between the solid and the liquid state. During fusion the temperature remains constant for constant pressure while the volume changes.

An *undercooled* or *supercooled liquid* is a liquid whose temperature is below the melting point without crystallization taking place.

A liquid may be hardened to an amorphous solid by undercooling, but in that case it has no definite freezing point.

The *heat of fusion* of a substance is the heat per unit mass necessary to melt the substance.

$$L = \frac{H}{m}.$$

Its *unit* in the metric system is the calorie-per-gram.

The heat of fusion is numerically equal to the heat needed to melt one gram of the substance. The equation $H = Lm$ furnishes another method for measuring heat.

$$[L] = [L^2 T^{-2}].$$

66. *Vaporization* is the transition from the solid or liquid state to the gaseous state.

The *vapor tension* of a substance is the pressure exerted by its saturated vapor upon the walls of the containing vessel.

The *vapor tension curve* is a curve on the temperature-pressure diagram representing the vapor tension of a liquid as a function of the temperature.

The vapor tension curve represents a stable equilibrium between the liquid and the gaseous state.

Evaporation is vaporization of a liquid under a pressure larger than its vapor tension.

It takes place from the free surface only.

Boiling is vaporization of a liquid under a pressure smaller than (rarely equal to) the vapor tension of the liquid.

It is accompanied by the formation of bubbles in the interior of the liquid.

The *boiling point* of a liquid is the temperature at which the vapor tension is equal to the external pressure.

The vapor tension curve therefore represents also the boiling point as a function of the external pressure.

The *normal boiling point* of a liquid is its boiling point under atmospheric pressure.

A *superheated liquid* is a liquid whose temperature is above the boiling point without boiling taking place.

The *heat of vaporization* of a substance is the heat per unit mass necessary to vaporize the substance :

$$L = H/m.$$

Its unit in the metric system is the calorie-per-gram.

The substance is usually understood to be at the normal boiling point.

$$[L] = [L^2T^{-2}].$$

67. *Sublimation* is vaporization of a solid.

The *sublimation curve* is a curve on the temperature-pressure diagram representing the vapor tension of the solid as a function of the temperature.

The sublimation curve represents an equilibrium between the solid and the gaseous state.

The *triple point* is the point on the temperature-pressure diagram where the fusion curve, the vapor tension curve and the sublimation curve meet each other.

At the triple point all three states are in equilibrium. While temperature and pressure are constant at the triple point the volume of a given mass is variable.

For water at the triple point: $t = + 0.0075^\circ \text{C.}$; $P = 4.58 \text{ mm of Hg.}$

68. The *dew point* is the temperature at which the water vapor, present in the air, becomes saturated. It is the temperature on the vapor tension or sublimation curve corresponding to the pressure of the water vapor present in the air.

Relative humidity is the ratio of the pressure of the water vapor in the air to the vapor tension of water at the temperature of the air.

69. The *critical point* is the point of greatest pressure or temperature on a diagram representing the region of mixture of the liquid and the gaseous states. It is the point at the upper end of the vapor tension curve.

The *critical temperature* is the temperature at the critical point. It is the temperature to which a gas must be cooled before liquefaction by pressure is possible.

Critical volume and *critical pressure* are the specific volume and the vapor tension at the critical point.

At the critical point the specific volumes of the liquid and the saturated vapor become equal.

70. *Convection of heat* is the transference of heat through space by means of discrete material bodies.

71. *Conduction of heat* is the transference of heat from particle to particle of a body or from one body to another in contact with it.

The *temperature gradient* at a point is the space rate of change of temperature at the point:

$$\kappa = \frac{dt}{dl}.$$

It is a vector quantity, taken in the direction of maximum change of temperature. If the temperature gradient between two points is uniform, it is the ratio of the difference of temperature between the points to the distance between them:

$$\kappa = \frac{t_1 - t_2}{l}.$$

$$[\kappa] = [L^{-1}\theta].$$

The *coefficient of thermal conductivity* (or *thermal conductivity, k*) is the time rate of heat conduction, $\frac{dH}{d\tau}$, per unit area divided by the temperature gradient:

$$k = \frac{dH}{Ad\tau} \cdot \frac{dl}{dt}.$$

If the temperature gradient is uniform and a steady condition has been reached,

$$k = \frac{Hl}{A\tau(t_1 - t_2)} \text{ and } H = k \frac{A(t_1 - t_2)}{l} \tau.$$

If the heat is expressed in calories, the temperature in degrees Centigrade, the distance in centimeters and the time in seconds, the coefficient of thermal conductivity is said to be expressed in *absolute units*. Frequently the conductivities are given as *relative conductivities* with respect to silver, water

or air. In such cases the coefficient of the standard is usually set equal to 100.

The coefficient of thermal conductivity is numerically equal to the heat transferred in unit time through unit area of a plate of unit thickness when unit difference of temperature is maintained between its faces.

$$[k] = [MLT^{-3}\theta^{-1}].$$

72. The *mechanical equivalent of heat* is the number of mechanical units of energy which equals the unit of heat.

It has no dimensions.

$$\begin{aligned} 1 \text{ calorie at } 15^{\circ} \text{ C.} &= 4.187 \times 10^7 \text{ ergs} = 4.187 \text{ joules,} \\ &= 427 \text{ gram-weight-meters.} \end{aligned}$$

$$1 \text{ British thermal unit} = 778 \text{ foot pounds.}$$

$$W \text{ ergs} = JH \text{ calories.}$$

$$\begin{aligned} J &= 4.187 \times 10^7 \text{ ergs per calorie,} \\ &= 778 \text{ foot pounds per B. T. U. etc.} \end{aligned}$$

73. An *isothermal process* is a process that takes place at a constant temperature.

An *adiabatic process* is a process that takes place without transference of heat to or from the system under consideration.

IV. MAGNETISM AND ELECTRICITY

(*n*) *Magnetism and Electricity* is that part of physics which treats of those phenomena which cannot be perceived directly by the senses and whose existence can be proved only by indirect methods.

Magnetic and electric phenomena are at present explained by the assumption of directional stresses in the ether and changes occurring in these stresses.

The existence of ether in space is accepted as a means of interpreting phenomena that cannot be explained by the properties of ordinary matter.

IV a. MAGNETOSTATICS

74. *Magnetic bodies or magnets* are bodies which attract or repel each other with a force which is neither gravitational nor due to mechanical action of ordinary matter, and which show a definite orientation relative to the earth's surface.

Magnetism is the name of a hypothetical substance producing attraction or repulsion between magnetic bodies by action at a distance.

The concept of magnetism as an imponderable fluid was introduced when the forces between magnets were supposed to be due to action-at-a-distance. It was assumed that magnetism is located on the surface of or within a magnetized bar on or near opposite ends.

Positive magnetism is the magnetism located on the north-seeking end of a magnetic needle; *negative magnetism* is the magnetism located on the south-seeking end of a magnetic needle.

75. *Magnetic poles* are the points of a magnet so situated that, if the positive and negative magnetisms were concentrated at these points, the magnetic action observed at some distance from the magnet would not be changed.

The *positive pole* is the north-seeking pole of a magnet, the *negative pole* the south-seeking pole.

The *strength of a magnetic pole*, m , is the quantity of magnetism at the pole. The force between two quantities of magnetism, m_1 and m_2 , is given by the equation:

$$F = \pm C \frac{m_1 m_2}{d^2}.$$

This force is explained as being due to a stress in the ether.

Unit pole strength, or *unit pole*, is that pole strength which repels a like pole of equal strength at a distance of one centimeter, in vacuo, with a force of one dyne.

Positive and negative poles appear always together in a magnet and are of equal strength.

76. The *permeability*, μ , of a substance is a property modifying the interaction of magnetic poles im-

mersed in, or separated by, this medium. In order to evaluate this constant the force of attraction or repulsion between two poles is taken as inversely proportional to the permeability:

$$F = \frac{1}{\mu} \frac{m_1 m_2}{d^2}.$$

The permeability is a fundamental unit and its dimensional expression is $[\mu]$.

This gives the dimensional formula for pole strength: $[m] = [M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1} \mu^{\frac{1}{2}}]$.

The *unit of permeability* is the permeability of the vacuum.

Diamagnetic bodies are bodies whose permeability is smaller than unity.

Paramagnetic bodies are bodies whose permeability is larger than unity.

Ferromagnetic bodies are bodies whose permeability is very large.

77. The *magnetic moment* of a magnet is the product of the strength of one pole and the distance between the poles: $M = ml$.

$$[M] = [M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1} \mu^{\frac{1}{2}}].$$

78. A *magnetostatic field*, or simply a *magnetic field*, is that space in which a magnet is acted upon by mechanical forces by virtue of its magnetic properties.

The *magnetic intensity*, H , of a field at a point is the property of the field giving rise to mechanical forces if a magnet be brought to the point. It is measured by the force per unit pole acting upon a pole brought to that point.

$$H = \frac{F}{m}.$$

It is a vector quantity, is numerically equal to and has the same direction as the force exerted at the point upon unit pole. It is independent of the presence of a magnet or of the existence of a force at the point. It is therefore to be regretted that this physical quantity is frequently called magnetic force. According to the ether-strain theory it is a measure of the magnetic stress.

Unit magnetic intensity is the intensity giving rise to a force of one dyne acting on unit pole. It is called the (*gauss*.) ~~oersted~~

$$[H] = [M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1} \mu^{-\frac{1}{2}}].$$

79. *Magnetic induction*, B , at a point is the product of the magnetic permeability μ of the medium into the magnetic intensity at the point.

$$B = \mu H.$$

It is a vector quantity, usually in the same direction as H . According to the ether-strain theory it is a measure of the magnetic strain.

Unit magnetic induction is the induction in a medium of unit permeability at a point where the intensity is one gauss. It has no generally accepted

name. For "line per square centimeter" and "maxwell per square centimeter" see below.

$$[B] = [M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1} \mu^{\frac{1}{2}}].$$

Lines of magnetic induction are imaginary lines in a magnetic field which are at every point in the direction of the induction. For purposes of measurement it is customary to represent unit induction by one line per square centimeter of a surface at right angles to the induction.

Lines of induction are always closed lines. Lines of intensity may be defined in a similar manner. Lines of force is an expression often used, but is applied as well to lines of induction as to lines of intensity, a practice creating considerable confusion.

The *magnetic flux*, Φ , through a surface is the surface integral of magnetic induction.

$$\Phi = \int B dA \cos \theta,$$

where θ is the angle between the normal to the surface and the lines of induction. If the field is uniform, the magnetic flux through a surface A at right angles to B is

$$\Phi = BA.$$

Unit magnetic flux is the flux through unit area at a point where the induction is unity. It is called the *maxwell*. Therefore we may use as the unit of magnetic induction the *maxwell per square centimeter*.

$$[\Phi] = [M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1} \mu^{\frac{1}{2}}].$$

A *tube of induction* is a tubular surface the elements of which consist wholly of lines of induction.

A *unit tube* of induction is a tube through every crossed section of which the flux is one maxwell.

80. *Magnetization of a body* is a phenomenon resulting from its introduction into a magnetic field of different permeability.

Magnetization is always accompanied by a redistribution of the magnetic induction in the field.

Induced poles are poles which have been assumed to exist in a magnetized body in order to explain the changes in the condition of the field due to the introduction of the body, without taking into account the difference of permeability.

The *magnetizing field intensity*, H_2 , at a point in a magnetized body is the field intensity resulting from the introduction of the body.

It is in general different from the intensity of the original field and may be found by the vector addition of the original intensity and the demagnetizing intensity due to the induced poles.

The *magnetic induction*, B_2 , in a magnetized body is the product of its permeability and the intensity of the magnetizing field. $B_2 = \mu_2 H_2$.

The *intensity of magnetization*, J , of a magnetized body is $\frac{1}{4\pi}$ times the difference between its mag-

netic induction and the induction which would exist if the permeability of the body were the same as the permeability μ_1 of the original field.

$$4 \pi J = B_2 - \mu_1 H_2 = B_2 \frac{\mu_2 - \mu_1}{\mu_2}.$$

If the intensity of magnetization is uniform, it is the magnetic moment of the body per unit volume. Intensity of magnetization is a physical quantity of the same nature as magnetic induction.

The *magnetic susceptibility*, κ , of a magnetized body is the ratio of its intensity of magnetization to the intensity of the magnetizing field.

$$\kappa = \frac{J}{H_2} = \frac{\mu_2 - \mu_1}{4 \pi}.$$

A magnetized body becomes a magnet if, upon the withdrawal of the external field, there remains a separate field whose lines of induction are linked with the body.

A *magnetization curve* is a curve representing the induction (or the intensity of magnetization) as a function of the magnetizing field starting with zero intensity in an unmagnetized specimen.

Magnetic hysteresis is a phenomenon arising from the transformation into heat of magnetic energy due to the tendency of a substance to persist in its state of magnetization.

A *hysteresis curve* is a curve representing the induction as a function of the magnetizing field when

the latter is carried through a complete cycle between equal positive and negative values.

81. *Magnetic potential*, Ω , is a physical quantity whose rate of decrease in any direction equals the component of the magnetic intensity in that direction.

$$H_L = -\frac{d\Omega}{dL}.$$

It is a scalar quantity.

A *magnetic equipotential surface* is a surface all points of which are at the same potential.

The direction of the intensity of the magnetic field at any point is normal to the equipotential surface passing through the point.

$$H = -\frac{d\Omega}{dn}.$$

$$[\Omega] = [M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}\mu^{-\frac{1}{2}}].$$

The *difference of magnetic potential* between two points is the product of the intensity of the magnetic field and the distance between the points measured along the line of intensity:

$$\Omega_1 - \Omega_2 = \int_1^2 H dl \cos \theta.$$

If the field is uniform:

$$\Omega_1 - \Omega_2 = Hl \cos \theta.$$

It is numerically equal to the work necessary to transfer unit pole from one point to the other.

Unit difference of magnetic potential is the difference of potential between two points, one centimeter apart, in a uniform field whose intensity is one gauss. It is sometimes called a *gilbert*.

Magnetomotive force is the difference of potential along a line of intensity from any point in the field back to the same point:

$$\text{M. M. F.} = \int H dl.$$

It is numerically equal to the work done in moving unit pole from a point in the field along a line of intensity back to the same point.

82. *Reluctance or magnetic resistance, \bar{R}* , between two surfaces is the ratio between the magnetic difference of potential and the magnetic flux.

Between two equipotential surfaces including the same flux:

$$\bar{R} = \frac{\Omega}{\Phi}.$$

In a uniform field of constant permeability:

$$\bar{R} = \frac{l}{\mu A}.$$

For a complete magnetic circuit:

$$\Phi = \frac{\text{M. M. F.}}{\Sigma \bar{R}}.$$

Unit reluctance is that reluctance in which a flux

of one maxwell is produced by a difference of potential of one gilbert. It is sometimes called an *oerstedt*.

$$[\bar{R}] = [L^{-1}\mu^{-1}].$$

83. *Magnetic energy* is an expression for energy measured in magnetic units. Thus the energy stored in a given volume of a tube of induction between cross sections of difference of potential Ω is

$$W = \frac{1}{8\pi} \Phi \Omega \quad (\text{if } \mu \text{ is constant}).$$

The energy per unit volume at a point in a magnetostatic field is

$$\frac{dW}{d\tau} = \frac{1}{8\pi} BH = \frac{1}{8\pi} \mu H^2.$$

The energy spent in moving a magnetic pole m between two points whose difference of potential is Ω , is

$$W = \Omega m.$$

IV b. ELECTROSTATICS

84. *Electrified or electrically charged bodies* are bodies that attract or repel each other with a force which is neither gravitational nor due to mechanical action of ordinary matter, and that show no definite orientation relative to the earth's surface.

. *Electricity or electric charge, q* , is the name of a hypothetical substance producing attraction or repulsion of electrified bodies by action-at-a-distance.

✓ The concept of electricity as an imponderable fluid was introduced when the forces between electrified bodies were supposed to be due to action-at-a-distance. It was assumed that electricity is located at the surface of, or within, an electrified body. For a definition of electricity by the ether-strain theory see art. 88.

✓ *Positive electricity* is the electricity produced on glass by rubbing it with silk; *negative electricity* is produced on hard rubber by rubbing it with wool.

The force between two quantities of electricity q_1 and q_2 , a distance d apart, is given by the equation :

$$F = \pm C \frac{q_1 q_2}{d^2}.$$

The force is explained as being due to a stress in the ether.

The *electrostatic unit of electricity* or *unit charge* is that quantity of electricity which repels an equal quantity at a distance of one centimeter in vacuo with a force of one dyne.

Electric surface density is quantity of electricity per unit area :

$$\sigma = \frac{dq}{dA}.$$

If the charge is uniformly distributed over the area A ,

$$\sigma = \frac{q}{A}.$$

85. The *dielectric constant* or *specific inductive capacity*, c , of a substance is the property modifying

the interaction of a system of electrified bodies immersed in, or separated by, this medium. In order to evaluate this constant the force of attraction or repulsion between two quantities of electricity is taken as inversely proportional to the dielectric constant :

$$F = \pm \frac{1}{c} \frac{q_1 q_2}{d^2}.$$

The dielectric constant is a fundamental unit and its dimensional expression is $[c]$. This gives the dimensional formula for electricity in the electrostatic system :

$$[q] = [M^{\frac{1}{2}} L^{\frac{3}{2}} T^{-1} c^{\frac{1}{2}}],$$

and for surface density :

$$[\sigma] = [M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1} c^{\frac{1}{2}}].$$

The dimensional formulæ for electrical quantities in the electromagnetic system will be given in arts. 93-104.

The *unit* of dielectric constant is the dielectric constant of the vacuum.

86. An *electrostatic*, or simply an *electric field*, E , is that space in which an electrified body is acted upon by a mechanical force by virtue of its electric properties.

The *intensity of an electric field* at a point is the

property of the field giving rise to a mechanical force if an electrified body be brought to the point.

It is measured by the ratio of the mechanical force acting at the point upon an electrified body, to the quantity of electricity on the body :

$$E = \frac{F}{q}.$$

$$[E] = [M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}c^{-\frac{1}{2}}].$$

Unit intensity of an electric field is the intensity giving rise to a force of one dyne acting on unit charge.

Intensity of an electric field is a vector quantity and is numerically equal to and has the same direction as the force exerted at the point upon unit quantity of electricity.

It is independent of the presence of electrified bodies, or forces existing at the point, and it is therefore unfortunate that it is sometimes called the electric force. According to the ether-strain theory it is a measure of the electric stress.

87. *Electric displacement* or *electrostatic induction*, D , at a point is $\frac{1}{4\pi}$ times the product of the dielectric constant c of the medium into the electric intensity at the point :

$$D = \frac{c}{4\pi} E.$$

It is a vector quantity in the same direction and sense as E . According to the ether-strain theory it is a measure of the electric strain. At the surface of a conductor D becomes equal to the surface density. Some authors define it as $D = cE$.

Unit electrostatic induction is the induction in a medium of unit dielectric constant at a point where the electric intensity is 4π units.

$$[D] = [M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-1}c^{\frac{1}{2}}].$$

Lines of electrical induction are imaginary lines in an electrical field which are at every point in the direction of the induction. For purposes of measurement it is customary to represent unit induction by one line per square centimeter of a surface at right angles to the direction of the induction.

Electric intensity may similarly be represented by lines of intensity.

Since the electric intensity at a distance r from a concentrated unit charge is $\frac{q}{cr^2}$ and the surface of a sphere of radius r is $4\pi r^2$, q lines of induction emanate from a positive charge q . Similarly q lines end in a charge $-q$.

The *electric flux* Π through a surface A is the surface integral of the electric induction.

$$\Pi = \int D dA \cos \theta$$

where θ is the angle between the normal to the sur-

face and the lines of induction. If the field is uniform, the electric flux through a surface A at right angles to D is

$$\Pi = DA.$$

Unit electric flux is the flux through unit area at a point where the displacement is unity.

$$[\Pi] = [M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}c^{\frac{1}{2}}].$$

A *tube of induction* is a tubular surface the elements of which consist wholly of lines of induction.

A *unit tube* of induction is a tube through every cross section of which the electric flux is unity.

88. An *electrical conductor* is a body which distributes electrical charges with great rapidity.

An *insulator* or *dielectric* is a body which distributes electrical charges with extreme slowness.

An electric conductor is unable to support in its interior an electric strain. In an electrostatic field the tubes of induction terminate on the surface of conductors. Free electricity may, therefore, be defined as a manifestation of a discontinuity of tubes of electric induction.

89. *Electrical potential* is a physical quantity whose rate of decrease in any direction equals the component of the electric intensity in that direction.

$$E_L = -\frac{dV}{dL}.$$

It is a scalar quantity.

An *electric equipotential surface* is a surface all points of which are at the same electric potential. The direction of the electric intensity at any point is normal to the equipotential surface through the point.

$$E = -\frac{dV}{dn}.$$

$$[V] = [M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}c^{-\frac{1}{2}}].$$

The *electrical difference of potential* between two points is the product of the electrical intensity and the distance between the points along the line of intensity :

$$V_1 - V_2 = \int E dL \cos \theta = \int \frac{F}{q} dL \cos \theta = \frac{W}{q}.$$

It is numerically equal to the work necessary to transfer unit quantity of electricity from one point to the other.

Unit difference of potential is the difference of potential requiring the expenditure of one erg for a transfer of an electrostatic unit quantity of electricity.

A definite numerical value can be given to the potential at a point by choosing some point as being at zero potential (earth or infinity).

90. The *electrical capacity of a conductor* is a property by virtue of which it can, when insulated, hold

a charge. It is measured by the ratio of the quantity of electricity on the conductor to its potential :

$$C = q/V.$$

It is numerically equal to the quantity of electricity charging the conductor from zero to unit potential.

$$[C] = [Lc].$$

For a spherical conductor of radius r , surrounded by a medium of dielectric constant c :

$$C = cr.$$

The *capacity of a condenser* is a property by virtue of which it can hold equal quantities of positive and negative electricity if its terminals are at different potentials. It is measured by the ratio of the positive quantity of electricity stored in the condenser to the difference of potential between the two terminals :

$$C = q/(V_1 - V_2).$$

It is numerically equal to the quantity of electricity charging the condenser to unit difference of potential.

According to the ether-strain theory the *capacity of a condenser* is a property by virtue of which an independent electric field can be established in the dielectric between the opposite surfaces of the condenser.

The capacity of a condenser is proportional to the specific inductive capacity of a substance used as the dielectric; and the specific inductive capacity, or dielectric constant, of a substance is therefore numerically equal to the ratio of the capac-

ity C_1 of a condenser with this substance as a dielectric to the capacity C of the condenser without any material dielectric between the plates. For all practical purposes air may be chosen as the standard of dielectric constant ($c = 1.00059$).

$$c = C_1/C, \text{ numerically.}$$

For a spherical condenser of radii r_1 and r_2 , the plates being separated by a medium of dielectric constant c ,

$$C = c \frac{r_1 r_2}{r_2 - r_1}.$$

For a plane parallel condenser filled with a dielectric of thickness d ,

$$C = \frac{c}{4\pi d} A \quad (\text{nearly}),$$

where A is the total area of the dielectric between the plates of the condenser.

The *electrostatic unit of capacity* is a capacity charged to one electrostatic unit of difference of potential by unit charge.

The *dielectric strength* of an insulator is the maximum difference of potential per centimeter thickness which the insulator can support without rupture.

91. *Electrical energy* is an expression for energy indicating merely that energy may be measured in electrical units. Thus the energy spent in moving a quantity of electricity from one point of a field to another is the product of the quantity and the difference of potential between the points:

$$W = \int q dV = q(V_2 - V_1) = \int q E dL \cos \theta = \int F dL \cos \theta.$$

The energy spent in charging a condenser is one half the product of the difference of potential into the quantity of electricity in the condenser :

$$W = \int q dV = \int CV dV = \frac{1}{2} CV^2 = \frac{1}{2} CQ^2.$$

The energy per unit volume at a point of an electrostatic field where the intensity is E and the displacement D ,

$$\frac{dW}{d\tau} = \frac{1}{2} ED = \frac{1}{8\pi} cE^2.$$

92. An *electric spark* is the sudden breaking down of a dielectric accompanied by a passage of electricity. *It is a discontinuous phenomenon.*

An *electric current* is the passage of electricity through a conductor. By the ether-strain theory it is explained as the disappearance of an electric strain in the dielectric surrounding the conductor. Strictly speaking this is a "conduction current." *Current strength, intensity of current*, or simply *current* is the time rate of transference of electricity :

$$I = \frac{dq}{dt}.$$

It is numerically equal to the quantity of electricity passing in unit time, if the current remains constant. It is a scalar

quantity, either positive or negative. It is positive if a positive quantity of electricity flows from a point of higher potential to one of lower potential, *i.e.*, in the same sense as the intensity of the electric field.

$$[I] = [M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-2}c^{\frac{1}{2}}].$$

Unit current is a current in which unit quantity of electricity is transferred in unit time, if the current remains constant.

Current density, i , at a point is the current per unit area at the point.

$$i = \frac{dI}{dA}.$$

It is a vector quantity having a direction normal to the area chosen, and the same sense as the current. It is usually taken normal to the equipotential surface at the point.

$$[i] = [M^{\frac{1}{2}}L^{-\frac{1}{2}}T^{-2}c^{\frac{1}{2}}].$$

A *convection current* is the passage of electricity through a nonconductor, such as air or free space. Its density at any point is

$$i' = \frac{dq}{d\tau} v,$$

where $\frac{dq}{d\tau}$ is the quantity per unit volume (*electric volume density*) and v the velocity of the charges.

A *displacement current* is a physical quantity of the same dimensions as a conduction current, but connected with a variation in time of an electrostatic field. It is measured by the time rate of variation of electric displacement. Its density at any point of the dielectric is

$$i'' = \frac{dD}{dt}$$

$$I'' = \int i'' dA = \int \frac{dD}{dt} dA.$$

IV c. ELECTROMAGNETISM

(o) *Electromagnetic phenomena* are phenomena in which energy appears to us both as electric and as magnetic energy and in which there exists a definite relation between electric and magnetic quantities.

93. An *electric current* is a physical quantity connected with a transmission and transformation of energy which is accompanied by the establishment of a magnetic field, not due to magnets.

An *electric conductor* (see also art. 88) is a body which can be used to restrict the transmission of energy by an electric current to a definite path.

An *electric conduction current* (see also art. 92) is an electric current, characterized by the appearance of heat in a conductor and the existence of a mag-

netic field whose lines of induction are closed lines about the axis of the conductor.

The current is said to flow through the conductor. In the case of a conductor of circular cross section the tubes of magnetic induction are circles inside and outside of the conductor. The heat appearing in the conductor is the equivalent of energy flowing into it from the surrounding space. The current is a scalar quantity, and may be either positive or negative according to the direction of the magnetic induction encircling it.

A conduction current, I , may be measured by the intensity of the magnetic field produced by it. The relation between these two physical quantities is given by Laplace's equation

$$dH = K \frac{I ds}{r^2} \sin \alpha,$$

which integrated for a circular current of radius r gives for the field intensity at the center

$$H = \frac{2\pi}{r} I,$$

and integrated for an infinitely long straight current, gives for the field intensity at a distance r from the axis of the current

$$H = \frac{2I}{r}.$$

$$\text{M. M. F.} = 4\pi I.$$

In a very long solenoid of N turns and length L

$$H = \frac{4\pi N}{L} I.$$

$$\text{M. M. F.} = 4\pi NI.$$

$$[I] = [M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-1} \mu^{-\frac{1}{2}}].$$

The *electromagnetic unit of current* is that current which, when passing through an arc of unit length in a circle of one centimeter radius, will produce at the center of the circle a magnetic field intensity of one gauss; or unit current is that current which, when flowing through a circle one centimeter radius will produce at its center a magnetic field whose intensity is 2π gauss. This unit is called the C. G. S. unit of current in order to distinguish it from the practical unit in common use (see art. 105). It is independent of the permeability of the medium surrounding the current.

A *constant current* is a current which produces a magnetic field whose intensity is constant at every point.

The heating of the conductor as well as other effects produced by a constant current take place at a constant time rate.

An *alternating current* is a current which varies periodically between equal positive and negative values.

A *sinusoidal current* is a current whose values plotted as a function of the time are represented by a sine curve

$$I_t = I_m \sin(\omega t + \theta).$$

Current density, i , at a point (see also art. 92) is the current per unit area at the point.

It is a vector quantity having a direction normal to the area chosen and the same sense as the current.

$$[i] = [M^{\frac{1}{2}} L^{-\frac{1}{2}} T^{-1} \mu^{-\frac{1}{2}}].$$

94. *Quantity of electricity* is the time integral of an electric current

$$Q = \int I dt,$$

or for constant current during the time t ,

$$Q = It.$$

It is an hypothetical fluid supposed to pass through a conductor carrying a current. It is identical in its nature with electrostatic quantity of electricity.

$$[Q] = [M^{\frac{1}{2}} L^{\frac{1}{2}} \mu^{-\frac{1}{2}}].$$

The *electromagnetic unit of quantity of electricity* is the quantity passing in one second if the current is unity. It is called the C. G. S. unit quantity of electricity (see also art. 105).

This unit is 3×10^{10} electrostatic units of electricity (art. 84).

95. The *electric resistance*, R , of a conductor is a characteristic property of the conductor by virtue of which the energy of an electric current is transformed into heat. It may be measured by the equation

$$H = \int I^2 R dt,$$

or, if the current is constant during the time t ,

$$H = I^2 R t.$$

The electric resistance of a given conductor at a constant temperature is a constant, *i.e.*, independent of the current (Ohm's law). It is sometimes called ohmic resistance.

$$[R] = [L T^{-1} \mu].$$

The *electromagnetic unit of resistance* is that resistance in which heat equivalent to one erg is produced by unit constant current in one second. It is called the C. G. S. unit of resistance (see also art. 105).

Resistivity or specific resistance, ρ , is the characteristic property of a substance upon which the resistance of a conductor formed of this material depends. For purposes of measurement its numerical value is defined by the equation:

$$\rho = RA/l,$$

where R is the resistance, A the cross section and l the length of the conductor.

It is numerically equal to the resistance between two opposite sides of a unit cube of the substance.

$$[\rho] = [L^2 T^{-1} \mu].$$

Electrical conductance, G , and *conductivity*, g , are the reciprocals of resistance and resistivity.

96. *Electrical difference of potential*, $V_1 - V_2$, be-

tween two points of a circuit is the energy per unit quantity of electricity spent during the passage of the quantity of electricity from one point to the other.

$$V_1 - V_2 = \frac{W}{Q}.$$

In case there is only an ohmic resistance between the two points,

$$V_1 - V_2 = IR.$$

In a resistance a positive current flows from a point of higher potential to a point of lower potential. This is equivalent to saying: There is a fall of potential over the resistance equal to IR .

$$[V] = [M^{\frac{1}{2}} L^{\frac{1}{2}} T^{-2} \mu^{\frac{1}{2}}].$$

This quantity is identical in its nature with electrostatic difference of potential. It is a scalar quantity, either positive or negative, according to the sense in which it is taken.

The *electromagnetic unit of difference of potential* is the difference of potential produced at the terminals of unit resistance carrying unit current (see also art. 105). This unit is equal to $\frac{1}{3 \times 10^{10}}$ electrostatic unit (art. 89).

Electric intensity at a point in a circuit is the space rate of decrease of potential at the point.

$$E = - \frac{dV}{dL},$$

$$i = gE.$$

97. *Electromotive force*, E. M. F., is a difference of potential, tending to produce an electric current.

The *electromotive force* of a machine is the difference of potential between its poles, when they are not connected by a conductor, *i.e.*, when it is on an open circuit.

Terminal potential difference of a machine is the difference of potential between its poles when the circuit is closed.

A *direct or impressed electromotive force* in a circuit is an E. M. F. causing an electric current to flow through a circuit.

It is recognized by the fact that in the portions of the circuit containing a direct E. M. F. there is a rise of potential or a fall of potential smaller than the IR drop due to the ohmic resistance.

A *counter electromotive force* is an E. M. F. tending to send a current through the circuit in the opposite sense to that of the existing current.

It is recognized by the fact that in the portions of the circuit containing a counter E. M. F. there is a fall of potential greater than that caused by the ohmic resistance. It is usually given a negative sign. In any circuit carrying an electric current the sum of all the E. M. F.'s, taken with their proper sign, equals the fall of potential over the ohmic resistance of the whole circuit.

98. A *thermoelectromotive force* is an electromotive force produced when the junction of two dissimilar

conductors is kept at a different temperature from that of the other ends of the conductors.

Thermoelectric coefficient is the ratio of a thermoelectromotive force to the corresponding difference of temperature :

$$V' = V/t_1 - t_2.$$

It is usually called *thermoelectric power*.

↓ A *Peltier electromotive force* is that portion of the thermo-E.M.F. which is localized at the junctions of the conductors.

↓ A *Thomson electromotive force* is that portion of the thermo-E. M. F. which is located in the conductors and is due to the difference of temperature along the conductors.

99. An *electrolyte* is a conductor that is decomposed by the passage of an electric current.

Electrolysis is the decomposition of an electrolyte.

The *electrochemical equivalent* of a substance is the ratio of the mass in grams deposited in an electrolytic cell by the passage of electricity, to the quantity of electricity :

$$Z = \frac{m}{Q} = Z_H \frac{\text{atomic mass}}{\text{valence}} = Z_H \times \text{chemical equivalent}$$

(see also art. 105 d).

Ions are electrified mass particles formed by a dissociation of molecules.

Cations are ions which travel with the current and appear at the negative terminal of an electrolytic cell; *anions* are ions which appear at the positive terminal of an electrolytic cell.

The *equivalent conductivity*, λ , of an electrolyte is the conductivity of one gram equivalent of the dissolved substance when electrolyzed between electrodes one centimeter apart. It is usually referred to one liter of the solution and is measured by the equation

$$\lambda = \frac{1000}{n} g,$$

where n is the number of gram equivalents in a liter and g the electric conductivity (art. 95).

Polarization of an electrolytic cell is the effect of a current characterized by the establishment of a counter electromotive force.

It is usually restricted to the polarization of metals immersed in a liquid.

/ 100. The capacity of a condenser is a property by virtue of which it can hold quantity of electricity if its terminals are at different potentials. It is measured by the ratio of the quantity of electricity to the difference of potential between the terminals of the condenser.

$$C = \frac{Q}{V_1 - V_2}.$$

It is numerically equal to the quantity of electricity stored when the difference of potential is unity.

$$[C] = [L^{-1}T^2\mu^{-1}].$$

In the case of a variable current the difference of potential of the condenser is at the time t

$$(V_1 - V_2)_t = \frac{Q_t}{C} = \frac{\int_0^t I dt}{C}.$$

This may be considered as a counter E. M. F. and therefore in a circuit containing a variable impressed E. M. F. E , a resistance R and a capacity C

$$E_t - \frac{\int I dt}{C} = IR.$$

If the impressed E. M. F. is harmonic: $E_t = E_m \sin \omega t$.

$$I_t = \frac{E_m}{\sqrt{R^2 + \frac{1}{C^2\omega^2}}} \sin\left(\omega t + \tan^{-1} \frac{1}{CR\omega}\right);$$

$$I_m = \frac{E_m}{\sqrt{R^2 + \frac{1}{C^2\omega^2}}}.$$

The *C. G. S. unit of capacity* in the electromagnetic system is a capacity holding unit C. G. S. quantity of electricity if the potential difference between its terminals is one C. G. S. unit (see also art. 105). This unit is equal to 9×10^{20} electrostatic units (art. 90).

101. *Reactance, X*, due to capacity is a quantity of the dimensions of a resistance which when multiplied by the maximum current I_m gives the maximum counter E. M. F. due to the condenser (*reactive or con-*

denser *E. M. F.*). In the case of a harmonic current $I_t = I_m \cos(\omega t + \theta)$ the reactance due to a capacity is

$$X = \frac{1}{C\omega}.$$

The *unit of reactance* is the same as the unit of resistance.

The *impedance*, Z , of a circuit is a quantity of the dimensions of a resistance which when multiplied by a maximum current I_m equals the maximum impressed *E. M. F.* In the case of an harmonic current the impedance of a circuit containing resistance R and capacity C is

$$Z = \sqrt{R^2 + \frac{1}{C^2\omega^2}}.$$

The *unit of impedance* is the same as the unit of resistance.

102. *Electromagnetic induction* is the effect produced in an electrical field by a variation of a magnetic field, or *vice versa*.

The *coefficient of selfinduction*, or the *selfinductance* of a coil or circuit, is the ratio of the magnetic flux through it (*coil flux*), due to its own current, to the current strength:

$$L = \frac{\Phi_1}{I_1}.$$

If there are N turns in a coil and Φ_1' is the average magnetic flux through each turn,

$$\Phi_1 = N\Phi_1',$$

$$L = N \frac{\Phi_1'}{I_1}.$$

$$[L] = [L\mu].$$

Unit selfinductance is that selfinductance in which unit flux is produced by unit current. Selfinductance is usually measured by the counter electromotive force induced in the circuit by a variation of the flux or the current :

$$E' = - \frac{d\Phi_1}{dt} = - L \frac{dI_1}{dt}.$$

Unit selfinductance may also be defined as that selfinductance in which unit difference of potential is induced when the current in the circuit varies by unit current per second (see also art. 105).

The selfinductance of a very long circular coil of cross section A and length l , having n turns per centimeter of its length and being in a medium of constant permeability μ , is

$$L = 4 \pi \mu A n^2 l.$$

103. *Reactance* due to selfinductance is a quantity of the dimensions of a resistance which when multiplied by the maximum current I_m equals the maximum counter E. M. F. due to the selfinductance (*inductive*

E. M. F.). In the case of an harmonic current the reactance due to a selfinductance L is

$$X = L\omega.$$

The *impedance* of a circuit containing resistance and selfinductance is in the case of an harmonic current

$$Z = \sqrt{R^2 + L^2\omega^2}.$$

$$E_m = I_m \sqrt{R^2 + L^2\omega^2}.$$

$$I_t = \frac{E_m}{\sqrt{R^2 + L^2\omega^2}} \sin \left(\omega t - \tan^{-1} \frac{L\omega}{R} \right).$$

In the case of a circuit containing resistance, capacity and selfinductance and carrying an harmonic current, the reactance is

$$X = L\omega - \frac{1}{C\omega},$$

and the impedance

$$Z = \sqrt{R^2 + \left(L\omega - \frac{1}{C\omega} \right)^2}.$$

104. The *coefficient of mutual induction* or the *mutual inductance* of two coils or circuits is the ratio of the magnetic flux through one of them produced by a current in the other, to the intensity of this current:

$$M = \frac{\Phi_2}{I_1}.$$

$$[M] = [L\mu].$$

Selfinductance and mutual inductance are the same physical quantity, viewed from a different standpoint.

Unit mutual inductance is that mutual inductance in which unit flux is produced in one of the circuits by unit current in the other. Mutual inductance is usually measured by the counter electromotive force induced in either circuit by a variation of the flux through it or of the current in the other.

$$E_2' = - \frac{d\Phi_2}{dt} = - M \frac{dI_1}{dt}.$$

Unit mutual inductance may therefore also be defined as that mutual inductance in which unit difference of potential is induced in one circuit when the current in the other varies by unit current per second (see also art. 105).

105. The practical electrical units in the electromagnetic system are:

a. The *unit of resistance* is a resistance equal to 10^9 C. G. S. units (art. 95). It is called an *ohm*.

The ohm has been chosen as the first primary electrical unit. In order to distinguish from the next unit it is also called a fundamental electrical unit.

"The *international ohm* is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 grams in mass, of a constant cross-sectional area, and of a length of 106.300 centimeters."

The designation "international" has been given to certain concrete standards which are within the errors of the most accurate modern measurements equal to the corresponding fundamental units and are used for ordinary electrical measurements.

b. The unit of electric current is a current equal to 10^{-1} C. G. S. unit (art. 93). It is called an *ampere*.

This unit was chosen as the second primary unit. All other units are derived from these primary units.

The *international ampere* is the unvarying electric current which when passed through a solution of silver nitrate in water deposits silver at the rate of 0.00111800 of a gram per second.

The silver nitrate solution must be prepared and the measurement be carried out in accordance with specifications given by the International Conference on Electrical Units and Standards.

c. Unit difference of potential is the difference of potential produced at the terminals of one ohm by a constant current of one ampere. It equals 10^8 C. G. S. units (art. 96). It is called a *volt*.

The *international volt* is the electric difference of potential which when steadily applied to a conductor whose resistance is one international ohm will produce a current of one international ampere.

For all practical purposes it is sufficiently well represented by the 1.0183th part of the E. M. F. of a Weston normal cell at 20°C . The E. M. F. of a Weston or Cadmium normal cell is given by

$$E = 1.0183 - 0.0000406(t^{\circ} - 20) - 0.00000095(t^{\circ} - 20)^2 \text{ volts.}$$

d. Unit quantity of electricity is the quantity flowing through a circuit in one second if the current is constant and equal to one ampere. It equals 10^{-1} C. G. S. unit. It is called a *coulomb*.

96,530 coulombs are necessary to liberate by electrolysis the mass of one chemical equivalent. This makes the electrochemical equivalent of hydrogen $z = 0.0001036$ gram per coulomb (art. 99).

e. The unit of capacity is the capacity of a condenser which is charged to a difference of potential of one volt by one coulomb. It is called a *farad*. A *microfarad* = 0.000001 farad. One farad equals 10^{-9} C. G. S. unit, a microfarad equals 10^{-15} C. G. S. unit (art. 100).

Since a C. G. S. electromagnetic unit equals 9×10^{20} electrostatic units, one microfarad equals 900,000 electrostatic units and the capacity of a parallel plate condenser (art. 90), expressed in microfarads, is nearly

$$C = \frac{1}{900,000} \frac{c}{4\pi d} A = \frac{885}{10^{10}} \frac{c}{d} A.$$

f. The unit of inductance is the inductance in a circuit in which the induced E. M. F. is one volt, while the inducing current varies at the rate of one ampere per second. It equals 10^9 C. G. S. units (art. 102). It is called a *henry*.

The henry is the unit for selfinductance as well as for mutual inductance. The selfinductance of a very long coil (art. 102) expressed in practical units becomes nearly

$$L = \frac{4\pi}{10^9} \mu A n^2 l.$$

g. The *unit of energy* is the energy expended by one coulomb in passing through a difference of potential of one volt. It equals 10^7 C. G. S. units. It is therefore identical with the *joule* and called by that name. The *kilowatt-hour*, equal to 3,600,000 joules, is frequently used.

Energy may be expressed in terms of electrical units in many different ways (see also art. 91). Thus the energy spent in a conductor carrying a current:

$$W = \int I^2 R dt,$$

or for constant current

$$W = I^2 R t = Q(V_1 - V_2).$$

Energy stored in a charged condenser:

$$W = \frac{1}{2} Q(V_1 - V_2).$$

Energy stored in the field when a current I is established through a coil of selfinductance L :

$$W = \frac{1}{2} L I^2.$$

Energy stored in a field of two circuits carrying currents I_1 and I_2 respectively :

$$W = \frac{1}{2} L_1 I_1^2 + L_2 I_2^2 + M I_1 I_2.$$

h. The *unit of power* is the rate of expenditure of energy represented by one ampere flowing through a difference of potential of one volt. It equals 10^7 C. G. S. units. It is therefore identical with the *watt* and is called by that name. The *kilowatt* is used more frequently in practice.

The *international watt* is the energy per second expended by an electric current of one international ampere under an electric pressure of one international volt.

IV d. ELECTRONS AND RADIOACTIVITY

106. *Ionization* of a gas is the breaking up of its molecules into particles charged positively and negatively (see Ions, art. 99).

By ionization the gas becomes a conductor of electricity.

An *electron* or *corpuscle* is the smallest negative electric charge known to exist independently.

$$\frac{e}{m} = \frac{e}{m_0} \left(1 - \frac{u^2}{V^2} \right)^{\frac{1}{2}},$$

where e is the charge of the electron, m its mass, u its velocity, and V the velocity of light.

$$\frac{e}{m_0} = 1.73 \times 10^7 \text{ electromagnetic units per gram.}$$

$$e = 4.85 \times 10^{-10} \text{ electrostatic unit.}$$

Its mass which is possibly only apparent is very small as compared with that of an ion.

Cathode rays are streams of electrons projected from the cathode of a highly evacuated tube during discharge.

Canal rays or π rays are positively charged ions appearing behind a perforated cathode of a vacuum tube during discharge.

Roentgen rays or X rays are uncharged rays produced by a sudden stopping (change of velocity) of cathode rays, or of electrons.

107. Radioactive substances are substances which possess the property of spontaneously emitting radiations capable of passing through substances opaque to ordinary light.

α -rays or α -particles are positively charged rays emitted from radioactive substances.

$$\frac{e}{m} = 5,070 \text{ electromagnetic units per gram.}$$

$$e = 9.3 \times 10^{-10} \text{ electrostatic unit.}$$

The α -ray has been proved to be a helium atom with two elementary charges. They are identical with ordinary canal rays.

β -rays are negatively charged rays of high penetrating power emitted from radioactive substances.

They are streams of electrons of high speed. They are identical with cathode rays, but have greater speed.

γ -rays are uncharged rays emitted from radioactive substances. They are identical with Roentgen rays.

δ -rays are slow speed electrons produced by *α -rays* when traversing through matter.

V. OPTICS

(*p*) *Optics* is that part of Physics which treats of the physical phenomena affecting the sense of sight, and of the laws governing these and other phenomena of the same physical nature.

108. *Light* is a periodic disturbance in space capable of affecting the sense of sight.

It has a constant speed in an isotropic medium

$$V = \lambda n,$$

where n is the frequency. It may be considered as a wave motion in the ether and λ be called the wave length of the light under consideration.

Radiation is the collective name given to wave motion (24) of the ether independent of the sensations produced.

It is now considered as being an electromagnetic phenomenon.

Intensity of radiation, I , at a point is the time rate of transmission of energy per unit area taken at right angles to the direction of propagation of energy.

$$\int I dA = \frac{dW}{dt} \text{ or } I = \frac{dW}{d\tau} v,$$

where $\frac{dW}{d\tau}$ is the volume density of energy at the point and v its velocity.

It is numerically equal to the energy transmitted in unit time through unit area at right angles to the direction of propagation if the energy is uniformly distributed through space.

109. A *luminous body* is an independent or primary source of light.

An *illuminated body* is a dependent or secondary source of light.

It is visible only when light from a luminous body falls upon it.

A *transparent body* is a body through which sources of light can be seen sharply defined.

It transmits light without absorption.

An *opaque body* is a body which does not transmit light.

110. A *ray of light* is a directed line along which energy of light is being propagated.

A *pencil of light* is a portion of space filled with energy of light and bounded by rays.

A *converging pencil* is a pencil in which the rays are directed towards a point.

A *diverging pencil* is a pencil in which the rays come from a point.

A *beam of light* is a pencil bounded by parallel rays.

All these definitions may be generalized so as to include all radiation.

111. An *image* is an optical figure resembling a given object and formed by the light coming from this object.

It is situated at the points toward which the rays converge or from which they diverge; the points are the centers of curvature of the wave fronts.

A *real image* is an image through which the rays actually pass.

It can be formed on a screen.

A *virtual image* is an image from which the rays do not actually proceed.

It cannot be formed upon a screen.

112. *Reflection of light* is the phenomenon of a change in direction of a ray of light when it strikes upon a surface and is thrown back into the medium from which it came.

The *angle of incidence*, in wave motion, is the angle between the impinging wave front and the surface. It may also be defined as the angle between the normal to the surface and the incoming ray.

The *angle of reflection* is the angle between the reflecting surface and the reflected wave front. It may also be defined as the angle between the normal to the surface and the reflected ray.

Except when parallel rays fall upon a plane surface the angle of incidence, angle of reflection and angle of refraction (art. 113) have no definite value unless attention is fixed upon a particular point on the surface.

Diffuse reflection is reflection without the formation of an image.

113. *Refraction* is the phenomenon of a change in the direction of a ray when it passes from one medium to another.

Angle of refraction is the angle between the surface of separation and the wave front in the second medium. It may also be defined as an angle between the normal to the surface and the refracted ray.

The *index of refraction* between two media is the ratio of the sine of the angle of incidence to the sine of the angle of refraction.

It differs for radiation of different wave length.

The *absolute index of refraction* of a medium is the index of refraction between the ether and this medium.

The ratio of the indices of refraction of two media equals the inverse ratio of the velocities of light in the two media.

$$\frac{n_1}{n_2} = \frac{V_2}{V_1} = n_{21}.$$

The *critical angle* is the angle of incidence whose sine equals the index of refraction: $\sin \theta = n$.

The critical angle is a real quantity only when n is smaller than one, i.e., if the angle of incidence lies in the optically denser medium.

Double refraction of light is the separation of a beam of light into two separate beams upon entering an anisotropic medium.

114. A *focus* is a point from which rays diverge or toward which they converge.

It may be real or virtual.

Conjugate foci are two points so located that, with a source of light at one, its image is at the other.

The *principal focus* of an optical instrument is the point at which rays, originally parallel to the axis, are focused.

It is the focus conjugate to an object or image in infinity.

115. The *principal planes* of an optical system are planes normal to the optical axis and so situated that to each point in one plane there corresponds a conjugate point in the other plane at the same distance from the axis.

The distances of the principal planes of a lens, of thickness d and index of refraction n , from the corresponding surfaces are for large radii of curvature R_2 and R_1 of the lens surfaces:

$$h_1 = \frac{R_1}{R_1 + R_2} \frac{d}{n};$$

$$h_2 = \frac{R_2}{R_1 + R_2} \frac{d}{n}.$$

The *principal points* of an optical system are the points of the optical axis where it passes through the principal planes.

The *focal lengths*, f_1 and f_2 , of an optical system are the distances of the principal foci on either side from the nearest principal point:

$$\frac{f_1}{p} + \frac{f_2}{q} = 1,$$

where p and q are the distances of the object and image respectively from the corresponding principal planes.

$$\frac{f_1}{f_2} = \frac{n_1}{n_2},$$

where n_1 and n_2 are the indices of refraction of the first and last medium.

If the optical system is surrounded by a single medium, *e.g.* air,

$$f_1 = f_2 = f$$

and

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}.$$

The introduction of the principal planes allows the use of formulæ derived for very thin lenses.

116. The *resolving power* of an optical instrument is its ability to produce two distinct separate images of two closely adjacent points of the object viewed.

For a prism whose base has a thickness t and whose rate of variation of index of refraction with wave length is $\frac{dn}{d\lambda}$,

$$t \frac{dn}{d\lambda} \text{ must be greater than } \frac{\lambda}{\Delta\lambda}.$$

For a grating of n lines, and a spectrum of the m th order,

$$mn \text{ must be greater than } \frac{\lambda}{\Delta\lambda}.$$

For a telescope with an objective of diameter D and two objects appearing under the visual angle ϕ ,

$$\phi \text{ must be greater than } 2.4 \frac{\lambda}{D} \left(\text{sometimes stated as } 1.2 \frac{\lambda}{D} \right).$$

Spherical aberration is the production of a distorted image of an object, due to the form of the surface or surfaces of the instrument producing the image.

117. Dispersion of light is the transformation, upon refraction, of a beam of light into a diverging pencil, or into separate beams, due to the variation of the index of refraction with wave length.

The *dispersive power of a substance* is the ratio of the angular separation due to dispersion of two selected rays to the deviation from the original direction of the mean ray between them.

Color is a subjective experience permitting a discrimination of different qualities of light.

It is caused by the difference in wave length of ether vibrations affecting the sense of sight.

In Physics color is frequently used synonymously with radiation of definite wave length.

118. A *spectrum* is a succession of colors produced by the spreading of a beam of light into components of different wave length.

Its meaning may be generalized so as to include all radiation.

Ultra-violet waves are ether disturbances of wave lengths shorter than those of light.

They may be studied by making use of their photographic effects.

Infra-red waves are ether disturbances of wave lengths longer than those of light.

They may be studied by making use of their thermal effects upon absorbing bodies.

Electric waves are ether disturbances of wave lengths very long as compared with those of light.

They may be studied by making use of their electromagnetic effects.

Selective absorption is absorption of the energy of waves of certain wave lengths out of the total energy of radiation present.

Anomalous dispersion is dispersion through a substance whose index of refraction does not increase continuously as the wave length decreases in the region of the spectrum considered.

This phenomenon is due to absorption bands in the spectrum. The order of colors in the spectrum differs from that found in normal dispersion through transparent media.

119. *Chromatic aberration* is the production of an image whose color differs from that of the object.

It is due to dispersion, i.e., to the variation of the index of refraction with wave length.

120. *Polarized radiation* is radiation whose intensity of transmission through an anisotropic medium depends upon the orientation of the medium.

Plane polarized light (radiation) is polarized light in which the vibrations of the ether appear to be in one direction only.

The *plane of polarization* is the particular plane of incidence in which the polarized light is most copiously reflected.

The electric vector of plane polarized ether vibrations is at right angles to the plane of polarization. The same is true for the light vector producing photographic effects or fluorescence.

Elliptically polarized light (radiation) is polarized light in which the vibrations of the ether appear to be compounded of two components of plane polarized light, at right angles to each other.

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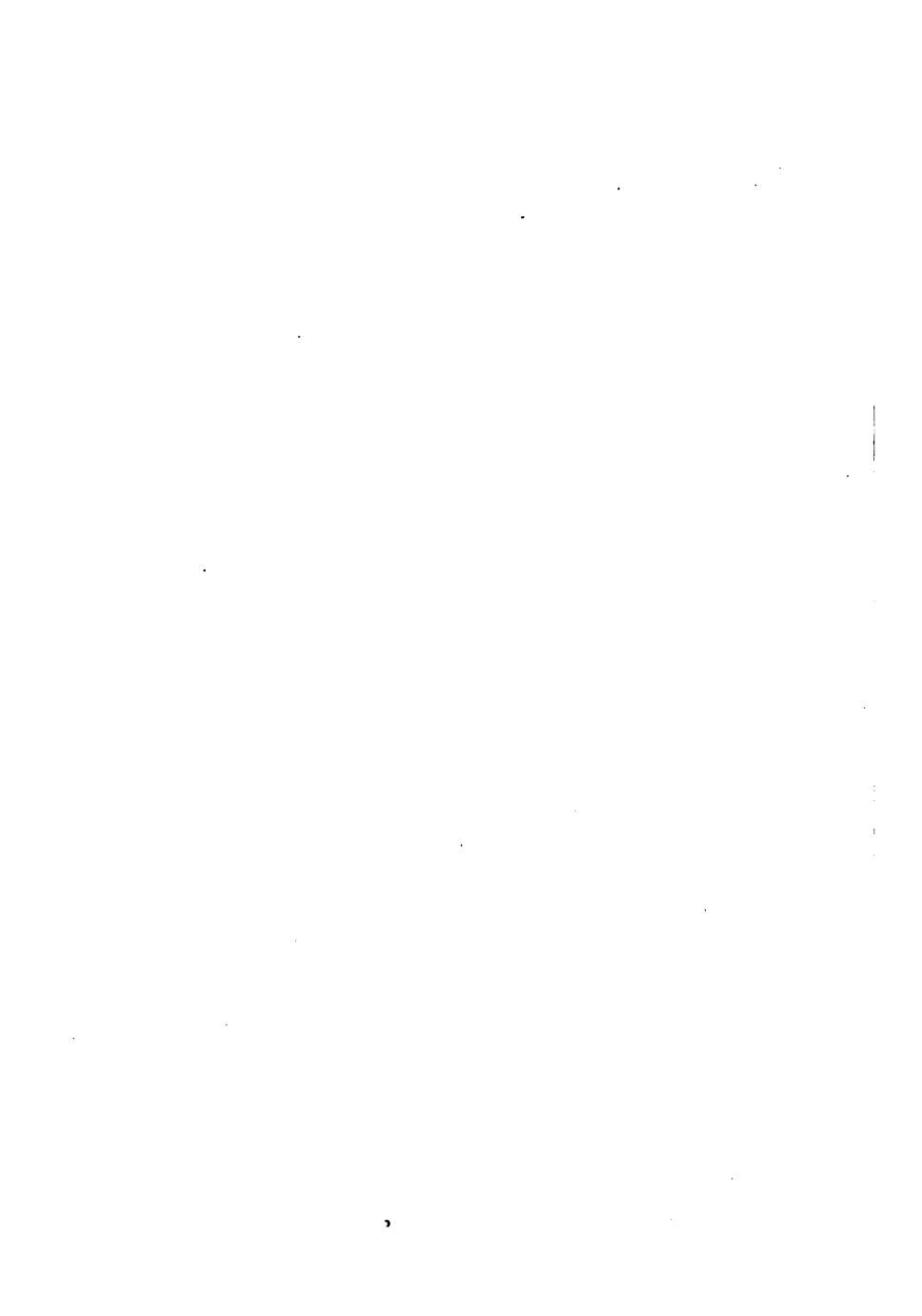
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